Funzione Descrittiva

SISTEMI CON SEMPLICI NON LINEARITÀ

- Le caratteristiche dipendono dall'ampiezza dei segnali → Non si applica la sovrapposizione
- Lo studio è molto più complesso
- Difficile anche imporre specifiche

Obiettivi minimi:

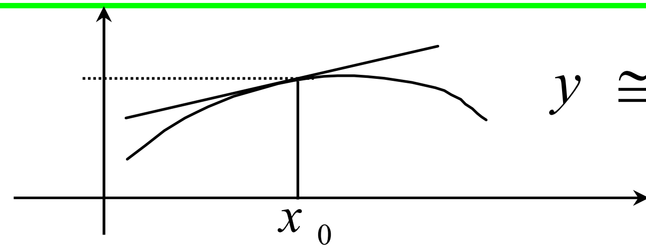
- Verificare la presenza di oscillazioni
- Calcolare i valori di regime

Ipotesi semplificative

- Una sola NL SISO nel loop
- La caratteristica è algebrica non differenziale → Non dipende da ω

(Rif. MARRO: Controlli Automatici)

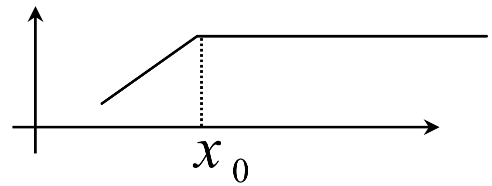
Taylor



$$y \cong y_0 + f'(x_0)\Delta x$$

Si trascurano le derivate >1

- esatta per $\Delta x \to 0$
- adatta $\forall \Delta x(t)$
- Cade se Δx grande, ad es. oscillazioni
- Non utilizzabile se $\nexists f'(x_0)$



Armonica

$x \sin(\omega t)$ → L → $Kx\sin(\omega t + \varphi)$

$x \sin(\omega t)$ → NL → $\Sigma Y_i(x)\sin[i\omega t + \varphi_i(x)] \cong$

$$\cong Y_0 + Y_1(x)\sin[\omega t + \varphi_1(x)]$$

Si trascurano le armoniche >1

- Non legata ad un punto di lavoro ma ad un tipo di ingresso
- Valida per ogni ampiezza

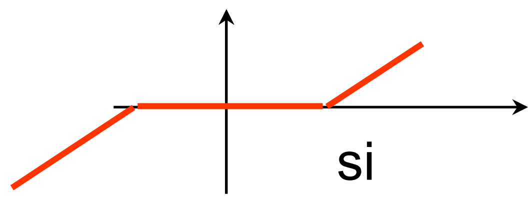
Sono entrambi sviluppi in serie di funzioni ortogonali

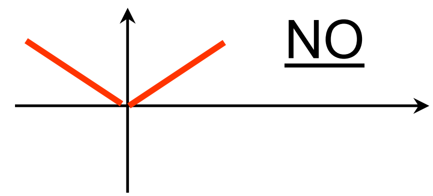
METODO DELLA FUNZIONE DESCRITTIVA

Ulteriori Ipotesi :

- Ingresso del sistema nullo
- Caratteristiche NL antisimmetriche

$Y_0 \equiv 0$





- la parte lineare è "passa basso" attenua le armoniche superiori

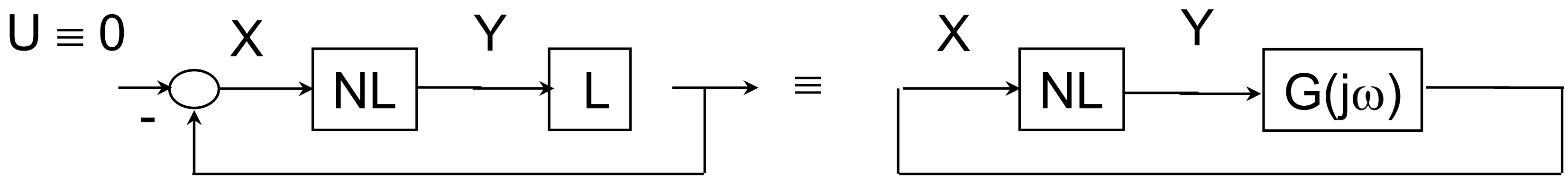
$$y(t) \cong Y_1(x) \sin\left[\omega t + \varphi_1(x)\right]$$

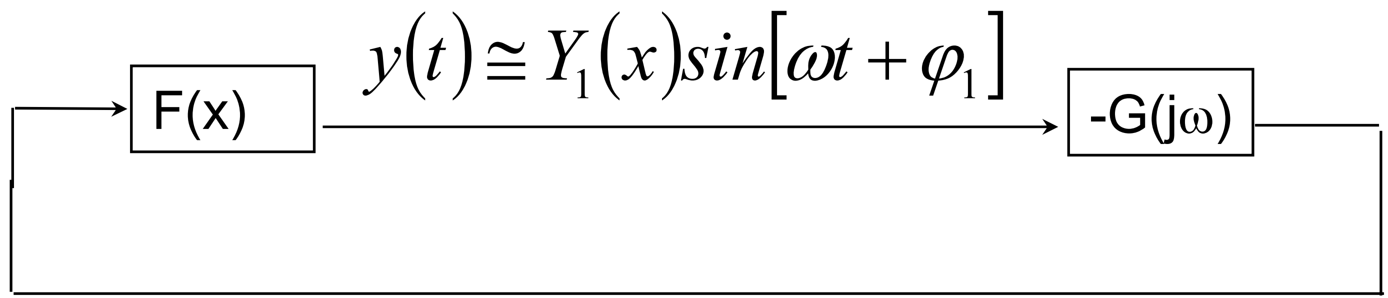
- Si definisce la "FUNZIONE DESCRITTIVA"

$$F(x) = \frac{1}{x} \cdot Y_1(x) \cdot e^{j\varphi_1(x)}$$

Complessa!

- Si approssimano le oscillazioni (cicli limite) con andamenti sinusoidali
- F(x) quasi una funzione di trasferimento, un guadagno variabile





$$x \sin \omega t \xrightarrow{F(x)} Y_1(x) sin\left[\omega t + \varphi_1\right] \xrightarrow{-G(j\,\omega)} x \sin \omega t$$

Condizioni per un ciclo limite $\bar{x}, \bar{\omega}$

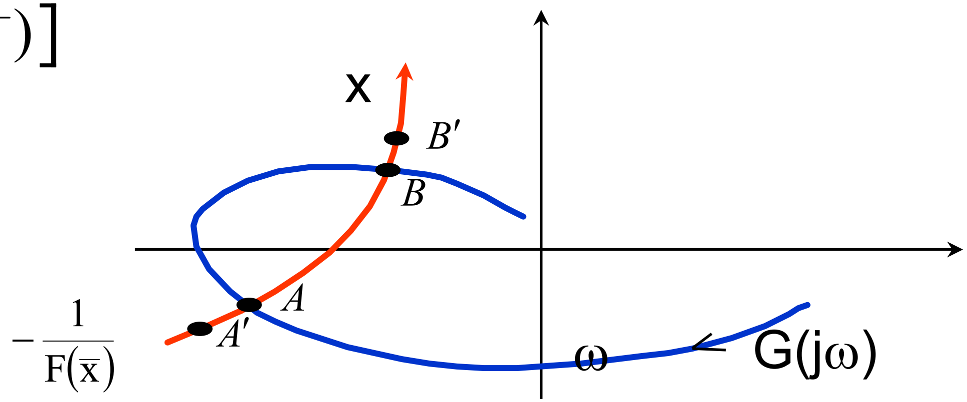
$$\begin{cases} |G(j\bar{\omega})| \cdot Y_1(\bar{x}) = \bar{x} \\ \angle G(j\omega) + \varphi_1(\bar{x}) = 180° \end{cases} \quad \Rightarrow \quad |G| = \frac{1}{|F|}$$

Interpretazione Grafica

$$\arg[G(j\overline{\omega})] = 180° - \arg[F(\overline{x})]$$

$$|G(J\overline{\omega})| = \frac{1}{|F(\overline{x})|}$$

$$-\frac{1}{F(\overline{x})} = G(j\overline{\omega})$$

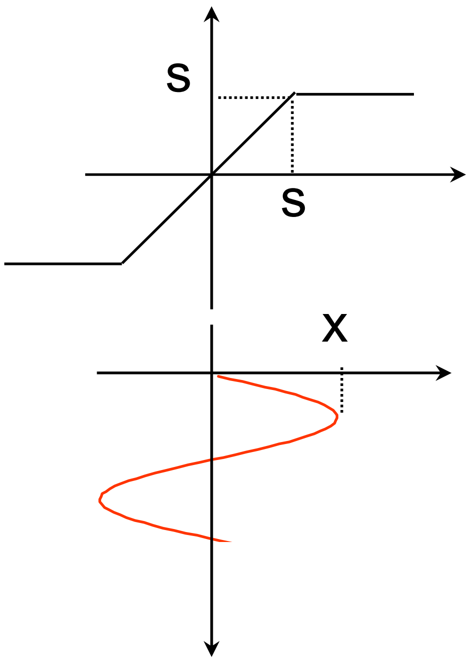


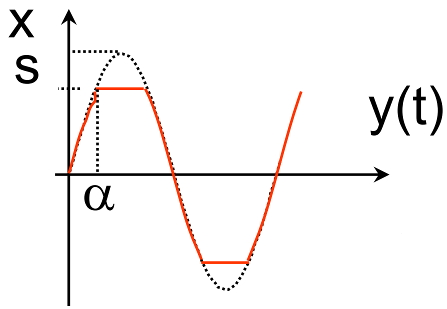
Quando c'è intersezione c'è un ciclo limite

A : instabile se $\overline{x}$ diminuisce (A')occorrerebbe|G|maggiore

B : stabile se $\overline{x}$ aumenta (B') con |G|=cost, può ridiminuire e tornare a B

Esempio di Calcolo (saturazione)





Se $x \le s$

$F(x) \equiv 1$

Occorre calcolare la 1ª armonica di y(t)

$$F(x) = \frac{1}{x}\left[b_1(x) + j a_1(x)\right]$$

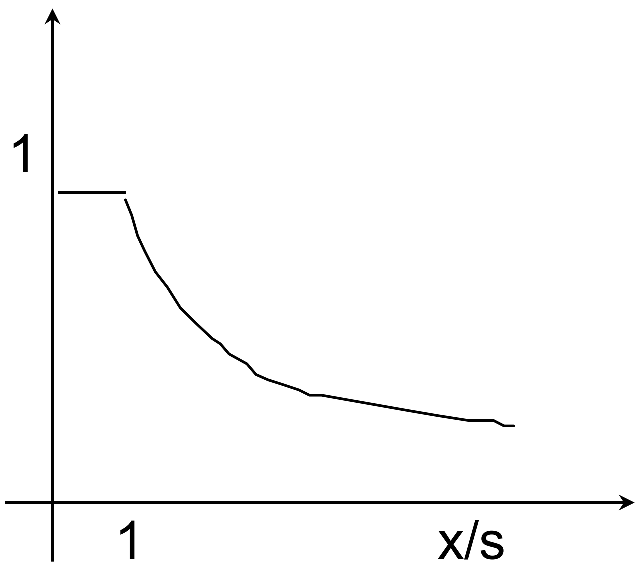
$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} y(t)\sin(\omega t)\, d\omega t$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} y(t)\cos(\omega t)\, d\omega t$$

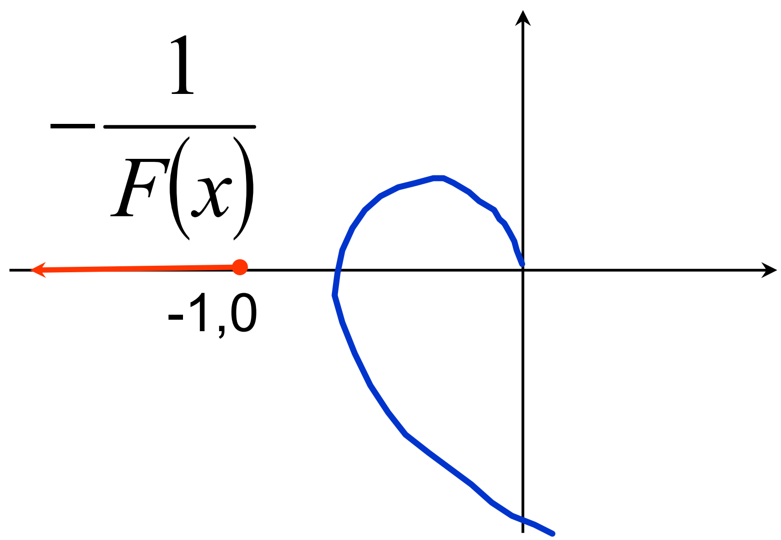
Per la simmetria $a_1 \equiv 0$ *F(x) solo Reale*

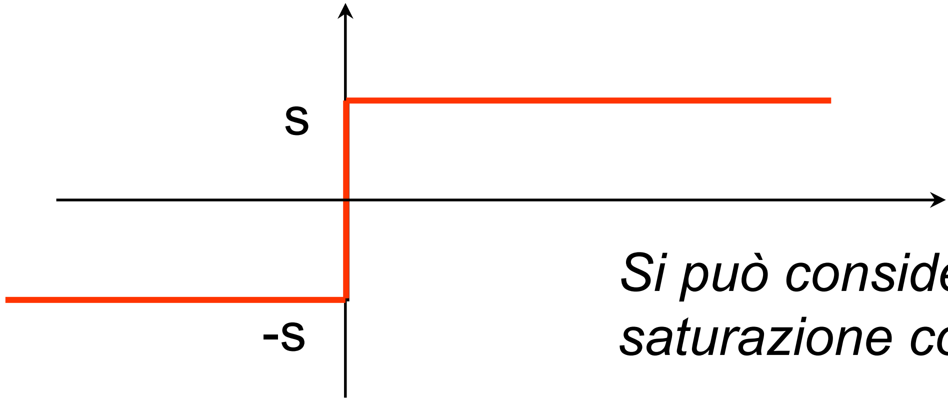
$$b_1 = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \cdots\cdots \quad = \frac{4}{\pi}\left[\int_0^{\alpha} x \cdot \sin^2 \omega t \cdot d\omega t + \int_{\alpha}^{\frac{\pi}{2}} s \cdot \sin \omega t \cdot d\omega t\right] \qquad \alpha = \sin^{-1}\left(\frac{s}{x}\right)$$

$$F(x) = \begin{cases} 1 & x \le s \\ \dfrac{2}{\pi}\left[\sin^{-1}\left(\dfrac{s}{x}\right) + \dfrac{s}{x}\sqrt{1-\left(\dfrac{s}{x}\right)^2}\right] & x > s \end{cases}$$



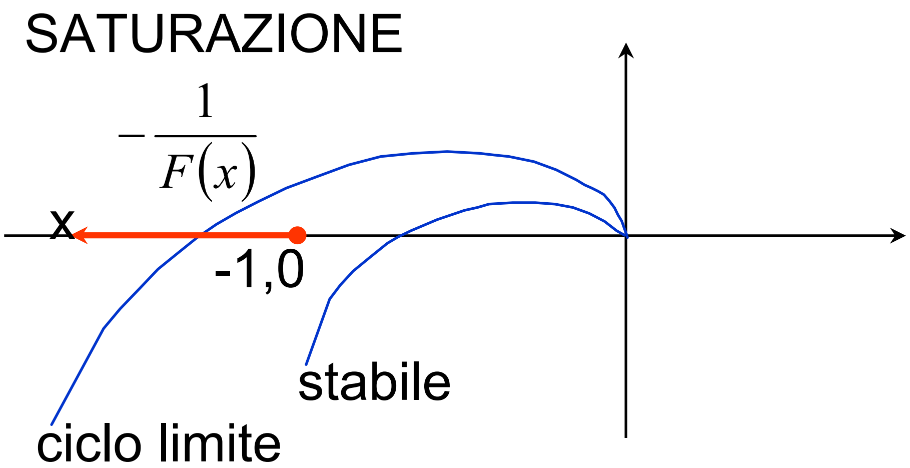
$$-\frac{1}{F(x)}$$

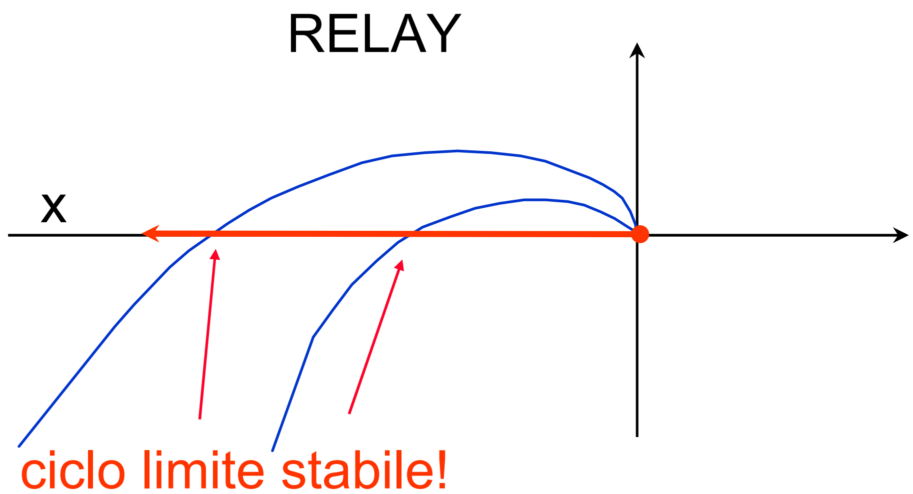




$$F(x) = \frac{4s}{\pi x}$$

Si può considerare come una saturazione con guadagno infinito

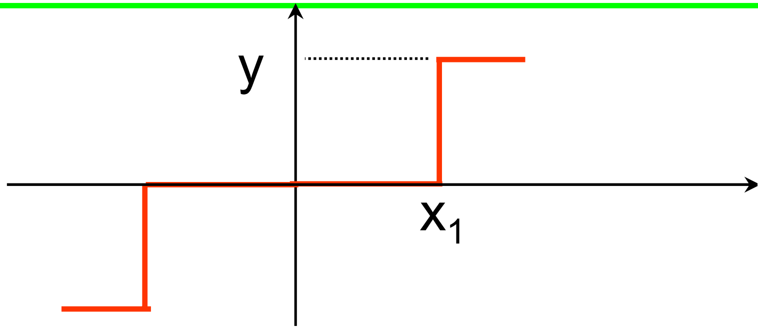


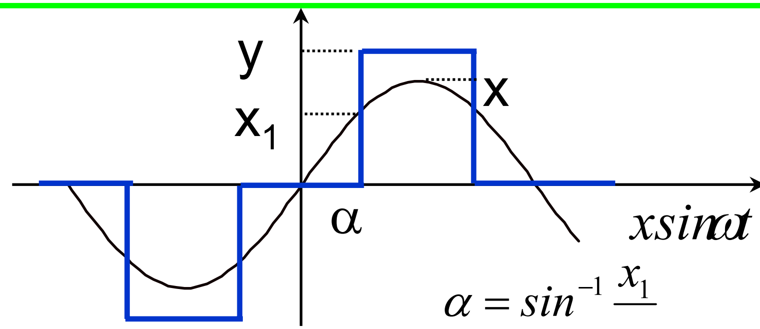


$$\begin{cases} \omega \rightarrow \infty \\ \bar{x} \rightarrow 0 \end{cases}$$

Si noti che per piccoli sfasamenti o $|G| \rightarrow 0$

Relay con soglia

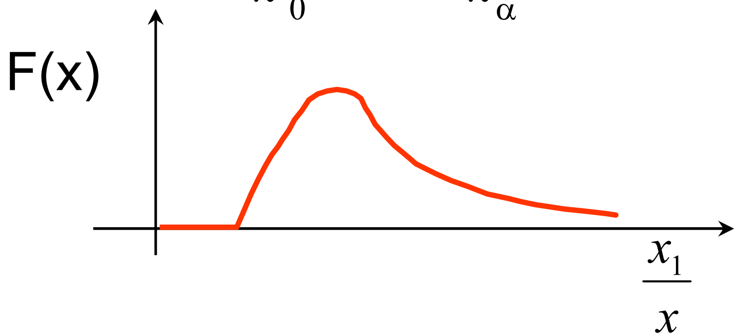


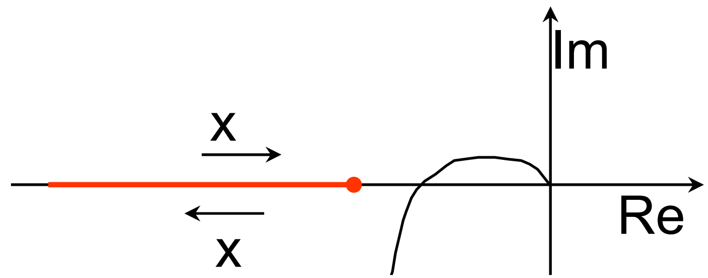


$$\alpha = \sin^{-1}\frac{x_1}{x}$$

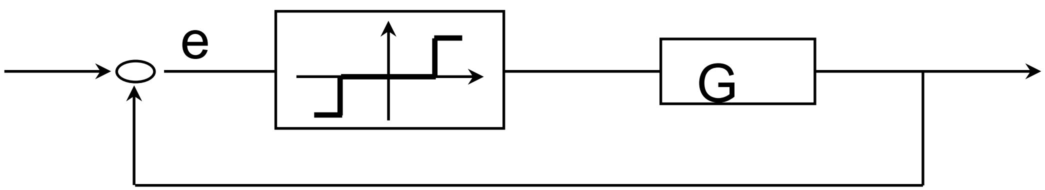
Simmetria $a_1 = 0$

$$b_1 = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} \ldots\ldots = \frac{4}{\pi}\int_\alpha^{\frac{\pi}{2}} Y_1 \cdot \sin\omega t \cdot d\omega t = \begin{cases} 0 & x \le x_1 \\ \dfrac{4Y_1}{\pi}\sqrt{1-\left(\dfrac{x_1}{x}\right)^2} & x > x_1 \end{cases}$$

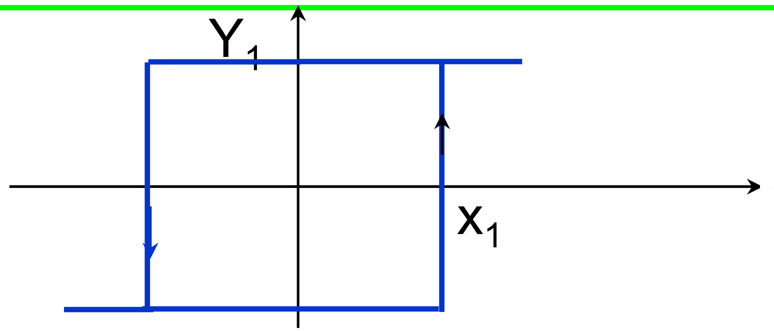




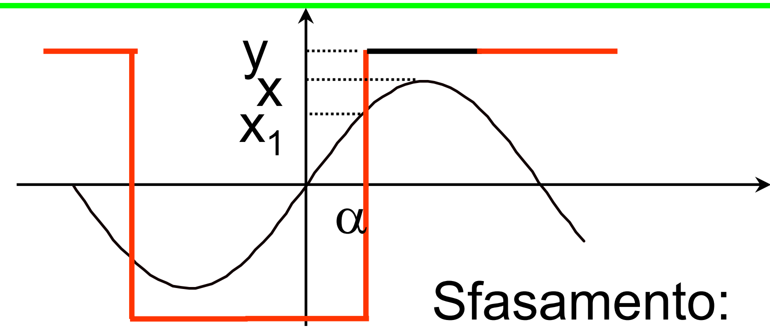
Due intersez. : un ciclo stabile
uno instabile



$-x_1 \le e \le x_1$
ciclo aperto!



F(x) valida per $x > x_1$

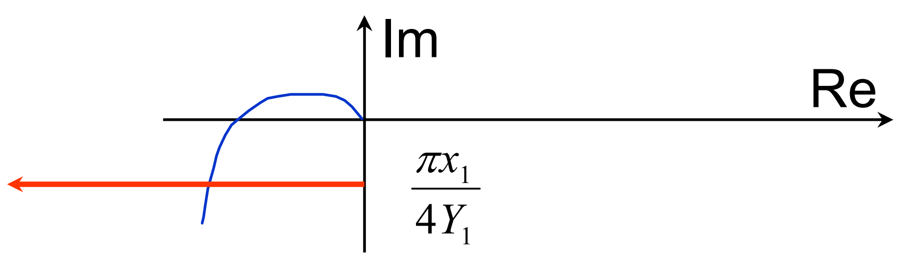


Sfasamento:
Im[F] ≠ 0

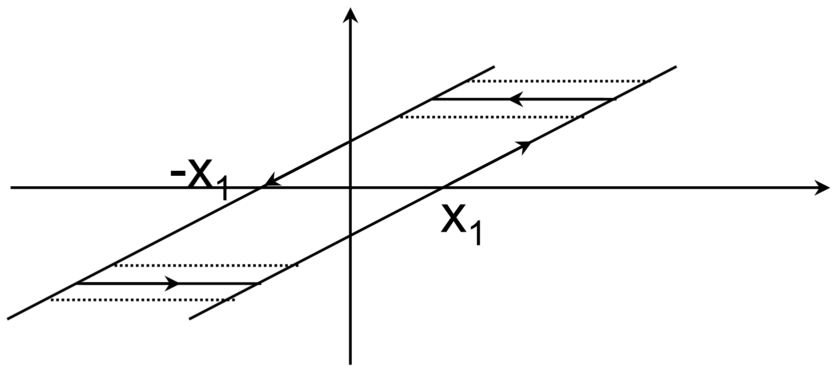
$$a_1 = -\frac{4Y_1}{\pi}\cdot\frac{x_1}{x} \quad , \quad b_1 = \frac{4Y_1}{\pi}\cdot\sqrt{1-\left(\frac{x_1}{x}\right)^2}$$

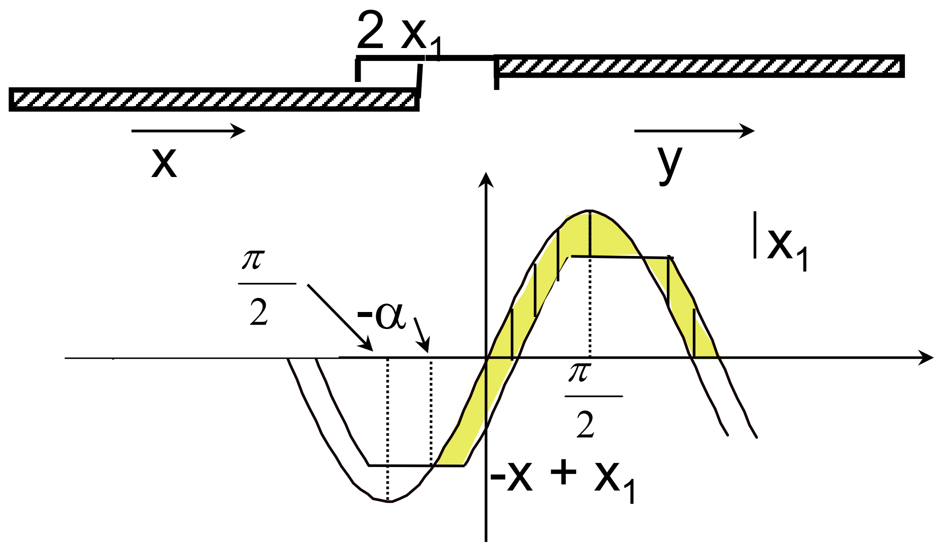
$$F(x) = \frac{1}{x}\left(b_1 + ja_1\right) = \frac{4Y_1}{\pi x}\left(\sqrt{1-k^2} - jk\right)$$

$$-\frac{1}{F(x)} = \frac{\pi x}{4Y_1}\left(\sqrt{1-k^2} + jk\right) = -\frac{\pi x}{4Y_1}\left[\sqrt{1-\left(\frac{x_1}{x}\right)^2} + j\frac{x_1}{x}\right]$$



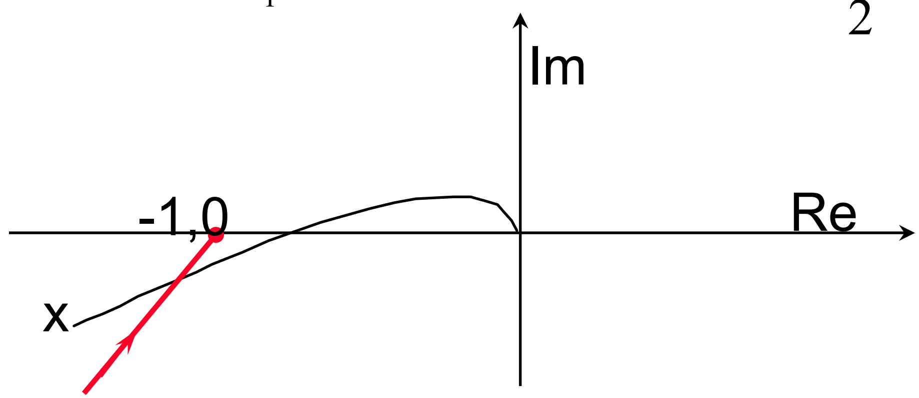
GIOCO



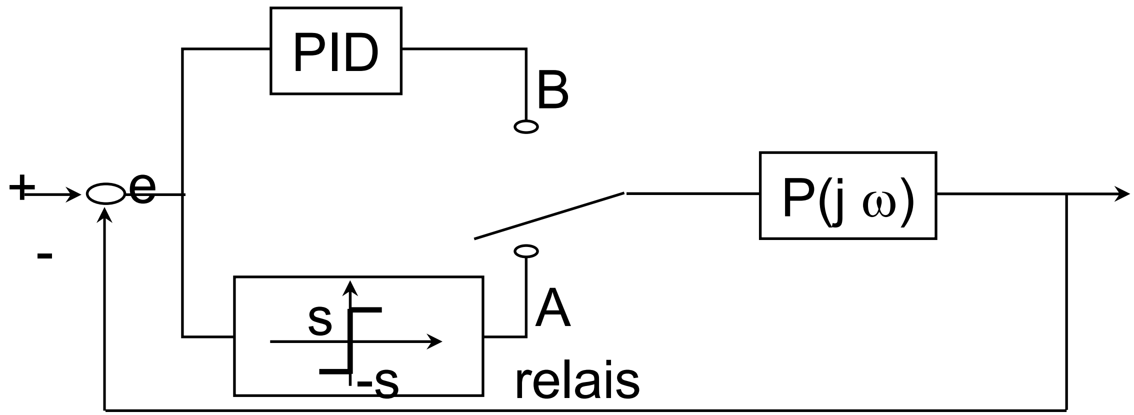


$$y(t) = -x + x_1 \qquad -\frac{\pi}{2} \le \omega t \le -\alpha$$

$$y(t) = x \cdot \sin \omega t - x_1 \qquad -\alpha \le \omega t \le \frac{\pi}{2}$$



A: il sistema oscilla
si misurano *e* e ω per
determinare ω_L e k_L



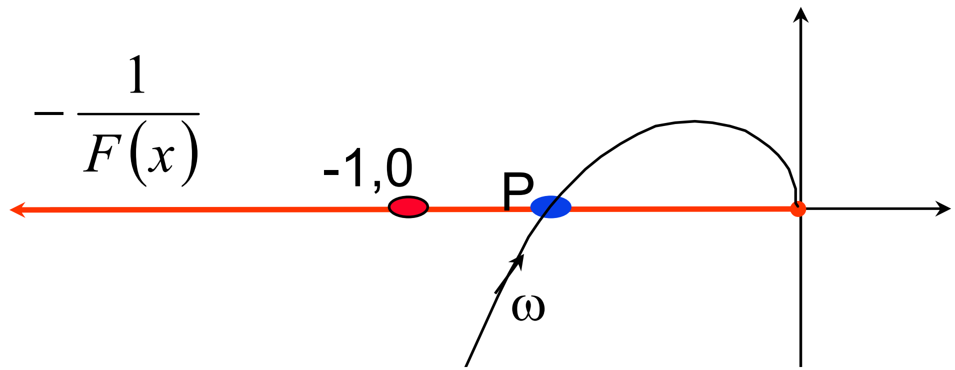
B: Normale
Funzionamento

$$F(x) = \frac{4S}{\pi x}$$

$$x \equiv | \; e \; |$$

$$\frac{1}{m_g} = \frac{\pi x}{4S} \quad , \quad \omega_L = \overline{\omega}$$

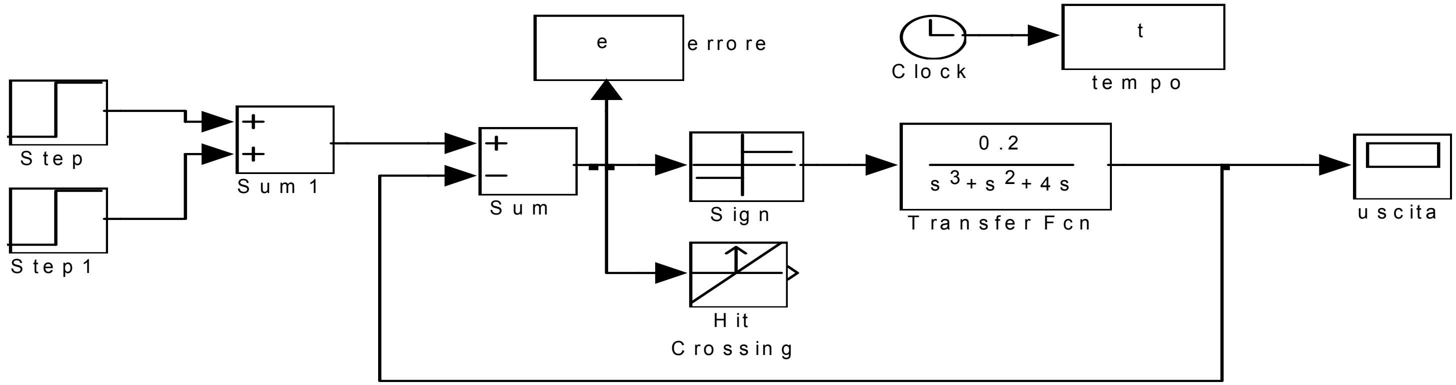
= margine di guadagno

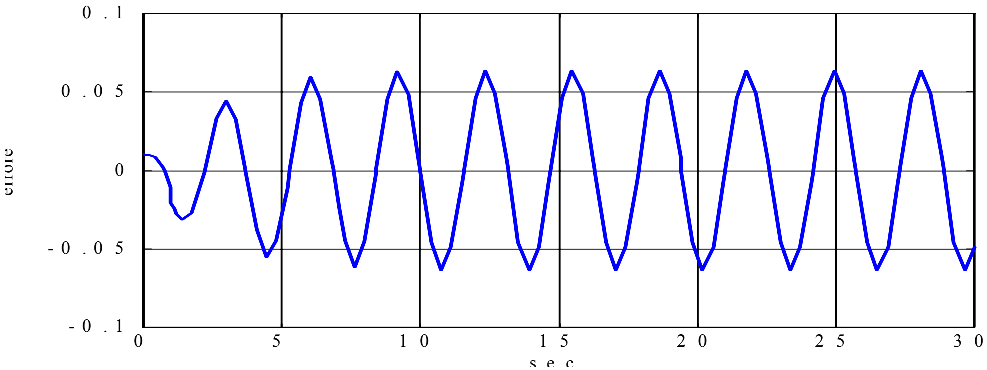


Si individua
la posizione di P
dalle misure

In base a questi valori si calcolano k_P , k_I , k_D

Poi si commuta sul PID



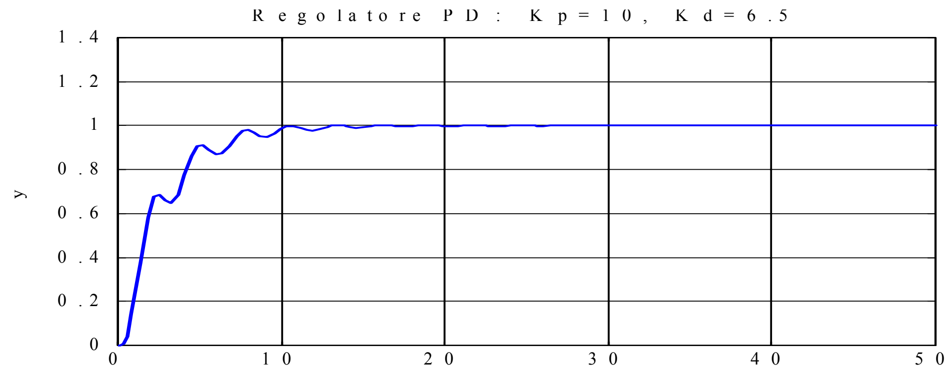


Con il relè:

$m_g=20$

$\omega_L=2.01$ rad/sec

Usando un regolatore Proporzionale Derivativo



Dai diagrammi di Bode e Nyquist: