

---

# TRASFORMATA DI FOURIER E LEGAME CON LA TRASFORMATA DI LAPLACE

SERIE DI FOURIER  
TRASFORMATA DI FOURIER  
TRASFORMATA DI LAPLACE  
CONFRONTO IN MAPLE

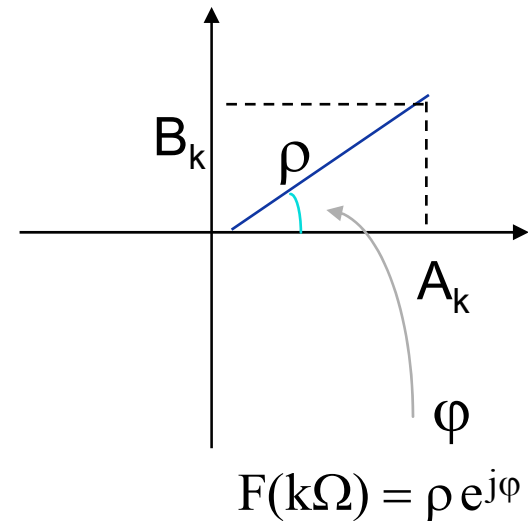
$f(t) = F(t \pm KT)$  periodica,  $T$ : periodo

$$f(t) = \frac{A_0}{2} + \sum_{k=1}^{\infty} \left[ A_k \cos \frac{2\pi k}{T} t + B_k \sin \frac{2\pi k}{T} t \right]$$

- $\frac{A_0}{2} = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt$  [valore medio]

- $A_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos \frac{2\pi k}{T} t dt$ ;

- $B_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin \frac{2\pi k}{T} t dt$

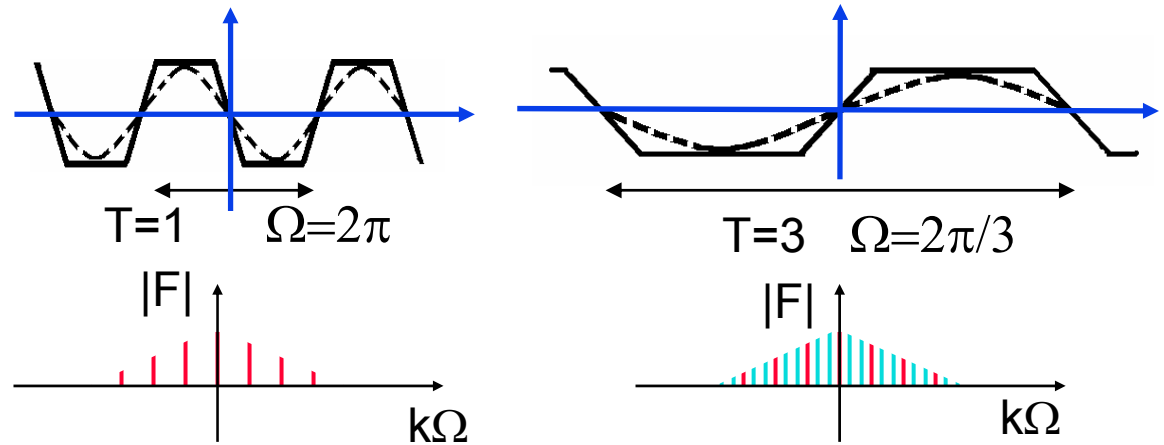


utilizzando i numeri complessi

$$\left[ \begin{aligned} f(t) &= \frac{\Omega}{2\pi} \sum_{k=-\infty}^{\infty} F(k\Omega) e^{jk\Omega t} \\ F(k\Omega) &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jk\Omega t} dt \end{aligned} \right] \quad \Omega = \frac{2\pi}{T}$$

# SERIE → TRASFORMATA

La serie è per segnali periodici, un segnale qualsiasi può essere visto come periodico con **periodo infinito**.



$$F(k\Omega) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jk\Omega t} dt$$

$$f(t) = \frac{\Omega}{2\pi} \sum_{k=-\infty}^{\infty} F(k\Omega) e^{jk\Omega t}$$

$$\Omega = \frac{2\pi}{T}$$

$$T \rightarrow \infty, \quad \Omega \rightarrow 0, \quad k\Omega \rightarrow \omega, \quad \sum \rightarrow \int \quad \rightarrow$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(t) = \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

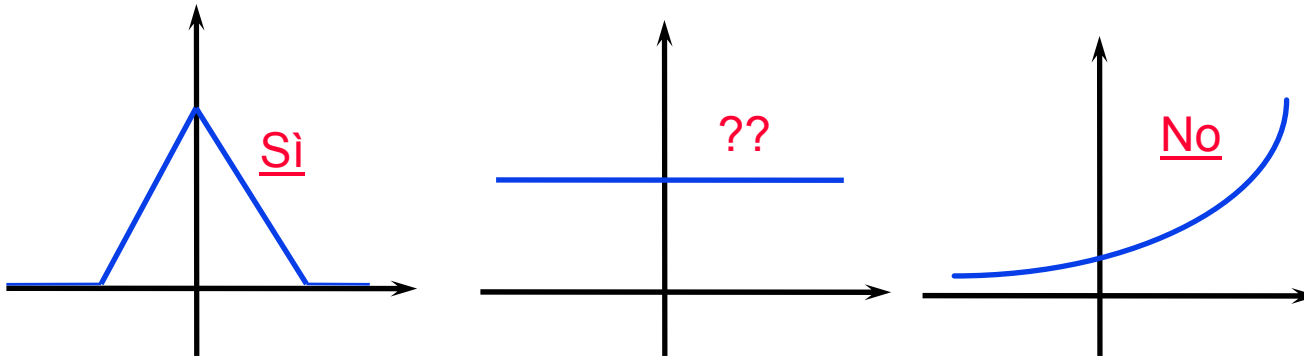
Condizione:  
f(t) sommabile

$$\int_{-\infty}^{\infty} |f(t)| dt < \infty$$

$$\left. \begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \\ F(t) &= \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega \end{aligned} \right\}$$

Corrispondenza  
biunivoca

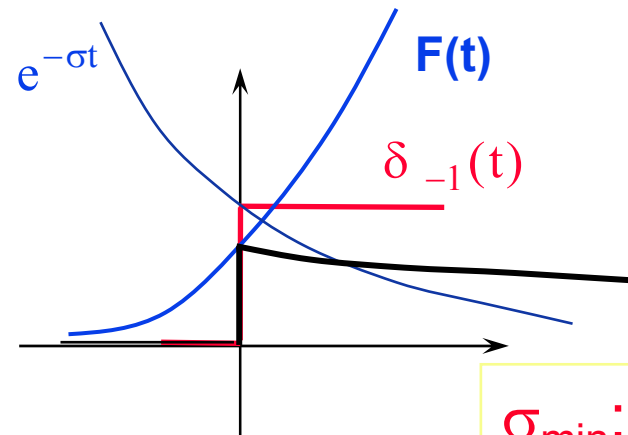
solo se  $\int_{-\infty}^{\infty} |f(t)| dt < \infty$



Sicchè proprio i casi più interessanti creano problemi

“TRUCCO” Invece di trasformare  $f(t)$ , trasformiamo:

$$\delta_{-1}(t)f(t)e^{-\sigma t}$$



Pongo  $s = \sigma + j\omega$

$$F(\sigma + j\omega) = \int_0^{\infty} f(t)e^{-\sigma t} \cdot e^{-j\omega t} dt$$

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

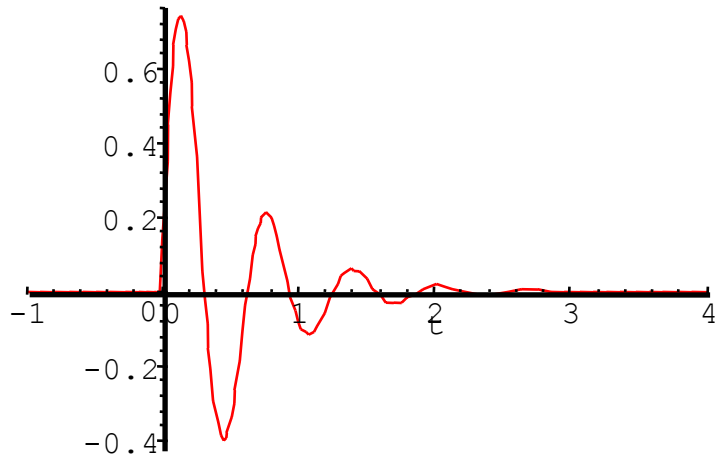
Trasformata  
di LAPLACE

# TRASFORMATA DI FOURIER IN MAPLE

```
> readlib(fourier);  
> sin(10*t)*exp(-2*t)*Heaviside(t);
```

$$\sin(10t) e^{-2t} \text{Heaviside}(t)$$

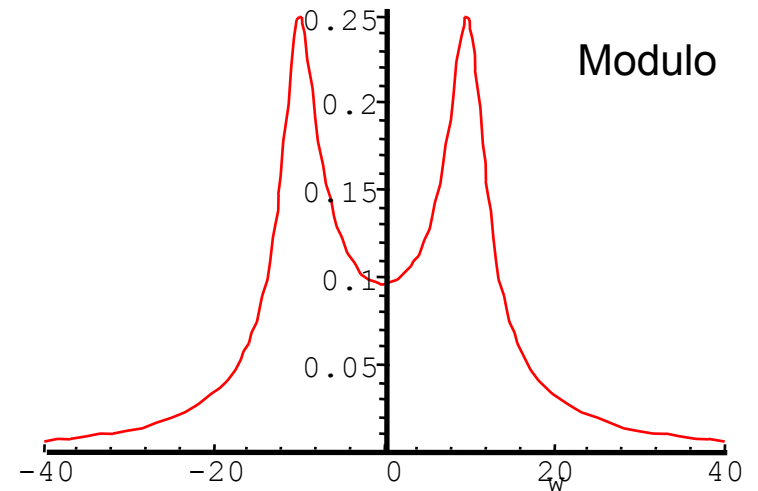
```
> plot(sin(10*t)*exp(-2*t)*Heaviside(t), t=-1..4, color='red');
```



```
> F:= fourier(sin(10*t)*exp(-2*t)*Heaviside(t), t, w);
```

$$F := -\frac{1}{2} \frac{I}{2 + Iw - 10I} + \frac{1}{2} \frac{I}{2 + Iw + 10I}$$

```
> plot(abs(F), w=-40..40, color='red');
```



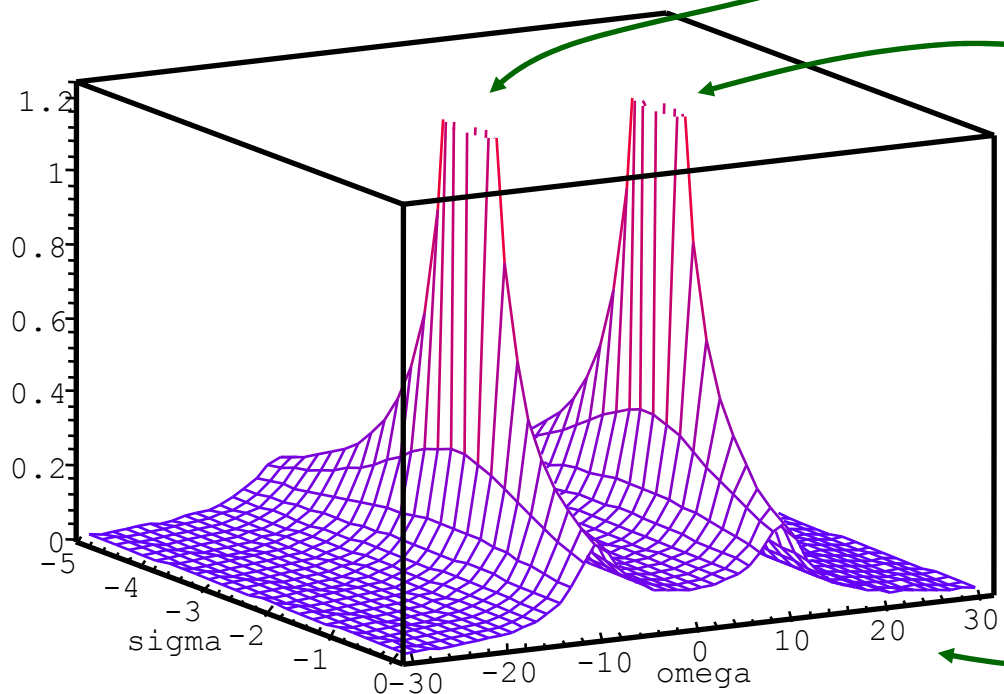
# TRASFORMATA DI LAPLACE IN MAPLE

> F:= laplace(exp(-2\*t)\*sin(10\*t),t,s);  $10 \frac{1}{s^2 + 4s + 104}$

> F1:= subs(s=sigma+I\*omega,F);  $F1 := 10 \frac{1}{(\sigma + I\omega + 2)^2 + 100}$

> plot3d( abs(F1), sigma=-5..0,omega=-30..30, view=0..1.25);

Grafico del Modulo  
(fase non riportata)



Effetto dei poli della  $F(s)$ . Il modulo va all'infinito

La sezione su questo piano è la risposta armonica (confronta con Fourier)

> Poli:= solve(denom(F)=0);  $Poli := -2 + 10I, -2 - 10I$