Risposta Armonica

(vedi Marro Par. 3.1 a 3.2)

(vedi Vitelli-Petternella par.VII.2, VII.2.1)

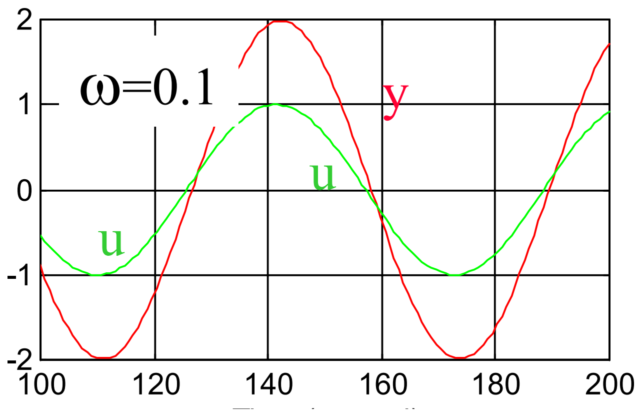
Che Cosa e'

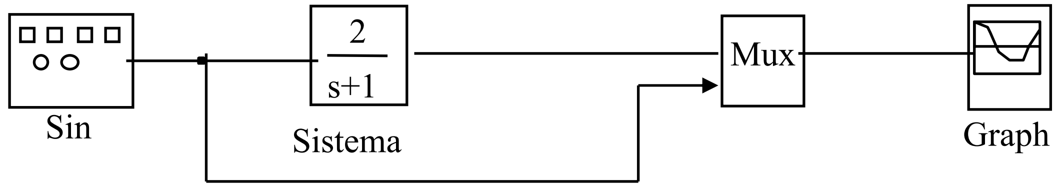
Come si calcola

(Come si misura)

Criteri di stabilita'

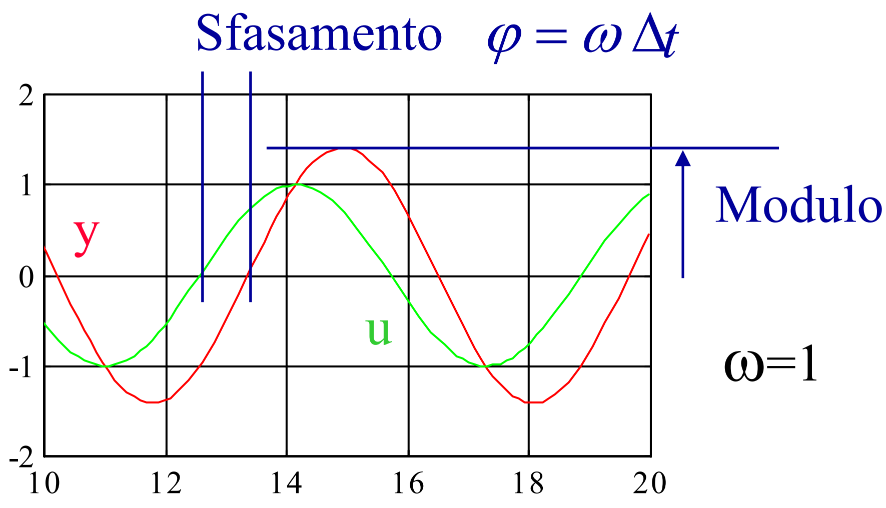
Che Cosa È (vedi BodeUnPolo_n.vi relizzato con Labview)

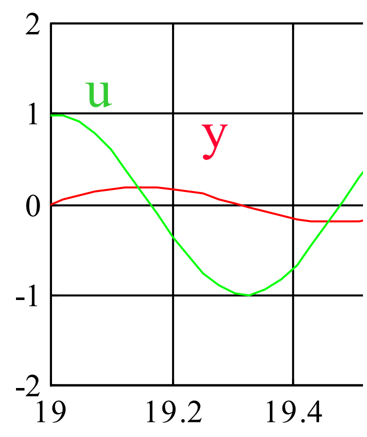




Risposta Armonica:

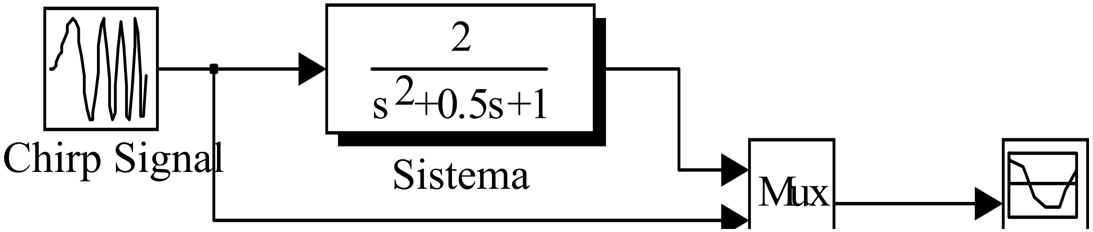
M(ω) e φ(ω) di Y(j ω)/U(j ω)



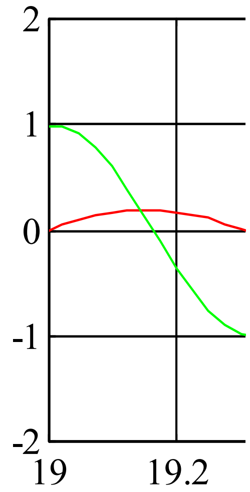


ω=10

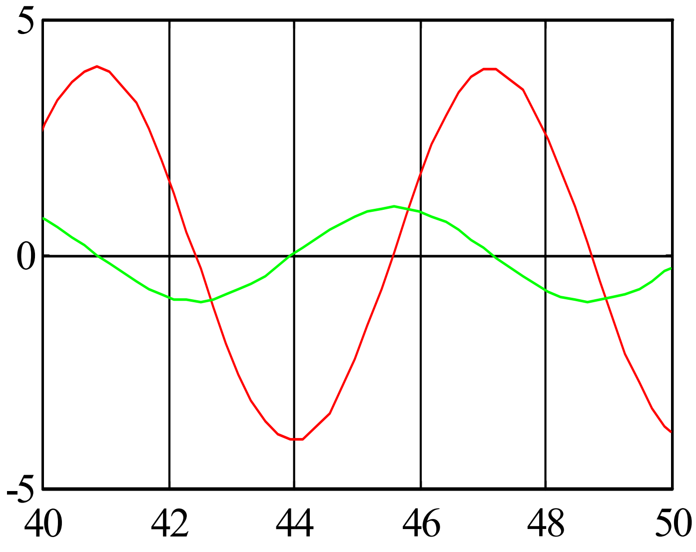
Alcune frequenze sono enfatizzate



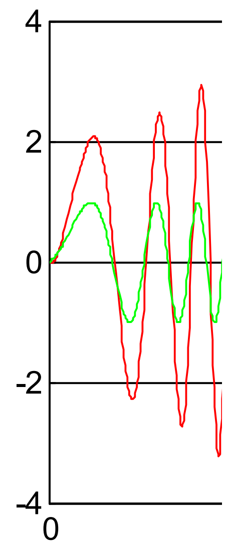
ω=10



ω=0.1



ω=0.06–2.4



- G(s) : asintoticamente stabile, Re $[p_i]<0 \quad \forall p_i$

$$u(t)\begin{cases}\sin\omega t & t>0\\ 0 & t\le 0\end{cases} \qquad U(s)=\frac{\omega}{s^2+\omega^2}$$

$$Y(s)=G(s)\cdot U(s)=G(s)\cdot\frac{\omega}{s^2+\omega^2}=G(s)\cdot\frac{\omega}{(s+j\omega)\cdot(s-j\omega)} = Y_t(s)+\underbrace{\frac{R}{s-j\omega}+\frac{R^*}{s+j\omega}}_{\text{permanente}}$$

Fattorizzazione

con $R=\lim_{s\to j\Omega}(s-j\omega)\cdot Y(s)=\lim G(s)\dfrac{\omega}{s+j\omega}=\dfrac{1}{2j}G(j\omega)$

$$y(t)=y_t(t)+\frac{1}{2j}\left[G(j\omega)\cdot e^{j\omega t}-G^*(j\omega)\cdot e^{-j\omega t}\right]=$$

$$=\cancel{y_t(t)} \quad + \quad \underbrace{\left|G(j\omega)\right|\cdot\sin(\omega t+\angle G(j\omega))}_{\text{permanente}}$$

Transitorio ⟶ 0

$$G(s)_{s=j\omega}=G(j\omega)$$

risposta armonica

Per G(s) razionale: $G(j\Omega) = \Sigma$ <u>tre</u> tipi di termini (e loro reciproci) quando

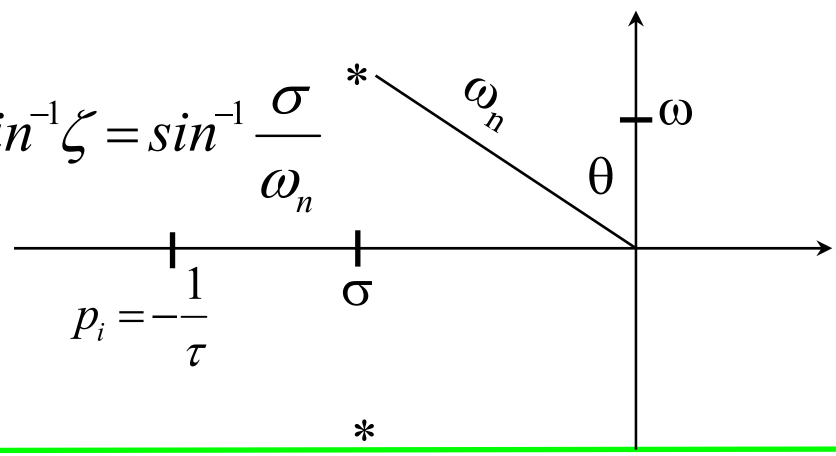
$$G(s) = \frac{\sum b_j s^j}{\sum a_i s^i} = \frac{N(s)}{D(s)}$$

Per ogni polinomio le radici possono essere:

		Termini della fattorizzazione	Notazione usuale
• $p_i = 0$	⟶	s	s
• p_i : reale	⟶	$(s-p_i)$	$1+\tau s$
• p_i, p_j coppia c.c.	⟶	$[s^2-2\sigma s+(\sigma^2+\omega^2)]$	$1+\frac{2\zeta}{\omega_n}s+\frac{s^2}{\omega_n^2}$

τ : costante di tempo
ω_n : pulsazione di risonanza
ζ : smorzamento, 0÷1

$$\theta = \sin^{-1}\zeta = \sin^{-1}\frac{\sigma}{\omega_n}$$



Scomposizione in termini semplici - 2

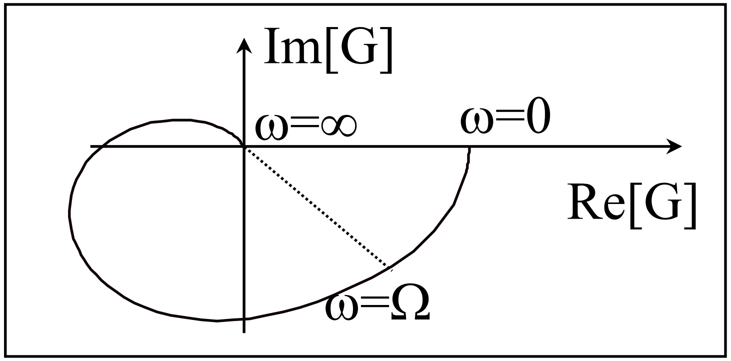
Comportamento asintotico G(jω):

$$G(s) = K \cdot \frac{\prod (1+\tau_i s) \prod \left(1 + \frac{2\zeta_i s}{\omega_{ni}} + \frac{s^2}{\omega_{ni}^2}\right)}{s^h \prod (1+\tau_j s) \prod \left(1 + \frac{2\zeta_j s}{\omega_{nj}} + \frac{s^2}{\omega_{nj}^2}\right)} = \frac{b_m s^m + \dots + b_0}{a_n s^n + \dots + a_0}$$

$j\omega \to 0 \;\cong t \to \infty$	$j\omega \to \infty \cong t \to 0$
$G(j\omega) \to \frac{K}{(j\omega)^h}$	$G(j\omega) \to \frac{b_m}{a_n}(j\omega)^{m-n}$
se $h = 0$: $G(j\omega) = k = \frac{b_0}{a_0}$	se $m < n$: $\frac{b_m}{a_n}\cdot\frac{1}{(j\omega)^{n-m}}$ $\quad |\cdot| \to 0 \quad \angle \to$ -(n-m)*90
se $h > 0$: $|G(j\omega)| \to \infty$, $\angle G(j\omega) \to -h \cdot 90^\circ$	$m = n$*: $\quad |\cdot| \to \frac{b_m}{a_n} \quad \angle\cdot \to 0^\circ$
	n>m non può accadere

* natura *"passa-basso"* dei sistemi fisici ⇔ CAUSALITA'

RAPPRESENTAZIONI DI G(Jω) (VEDI ES. REALIZZATI IN SCILAB)

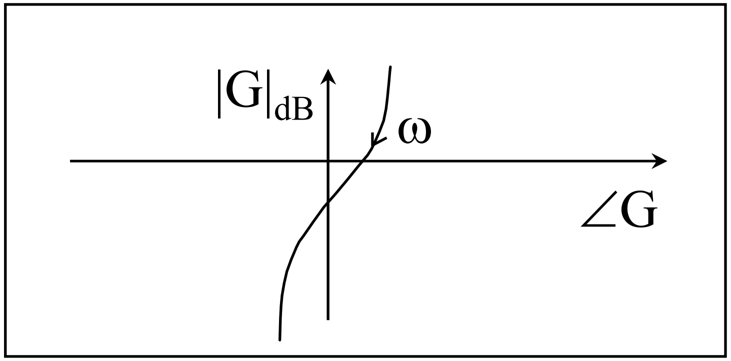


(vedi NumDenPoli_n.vi realizzato in LabView)

Polare o Re-Im (parametro=ω) : <u>NYQUIST</u>

(vedi Marro par. 3.5,vedi Vitelli-Petternella par.VIII.3)

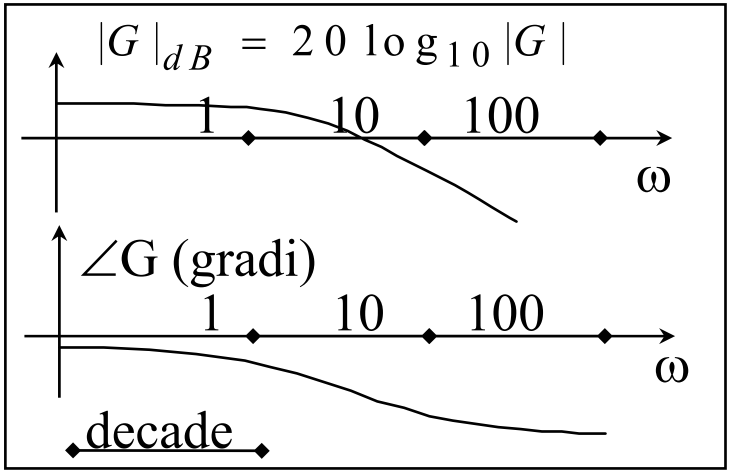
- Poco usato in pratica, utile in certe dimostrazioni.



Modulo-fase (parametro=ω) : <u>NICHOLS</u>

(vedi Marro par. 3.6, vedi Vitelli-Petternella par.VIII.2)

- Un tempo usata la carta di Nichols nella progettazione, oggi non più.



Modulo e fase separati : <u>BODE</u>

(vedi Marro par. 3.3, vedi Vitelli-Petternella par.VII.3)

- Ancora estremamente diffuso
- Usato nella strumentazione, come output dei calcolatori
- Usato nella progettazione manuale ed assistita

DECIBEL

•Bell : $B = \log_{10} \frac{W_2}{W_1}$ •Decibel : $dB = 10 \log_{10} \frac{A_2}{A_1}$

Poiché $W \propto V^2$, I^2 ..... *Es.:* $W=RI^2$ W : potenza quando si usano le ampiezze :

Decibel: $dB = 20 \log_{10} \frac{A_2}{A_1}$

Vantaggi:
- campo di valori maggiore a parità d'ingombro
- errore di rappresentazione costante in percentuale
- alcuni andamenti si semplificano

Alcuni valori

$\left. 2 \right|_{dB} \cong 6$

$\left. 0.1 \right|_{dB} = -20$

$\left. 10 \right|_{dB} = 20$

$\left. 20 \right|_{dB} \cong 26$

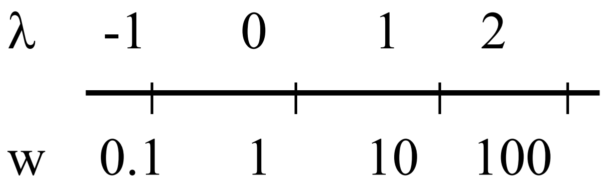
$\left| \frac{1}{G(j\omega)} \right|_{dB} = - \left| G(j\omega) \right|_{dB}$

$\left. 1 \right|_{dB} = 0$

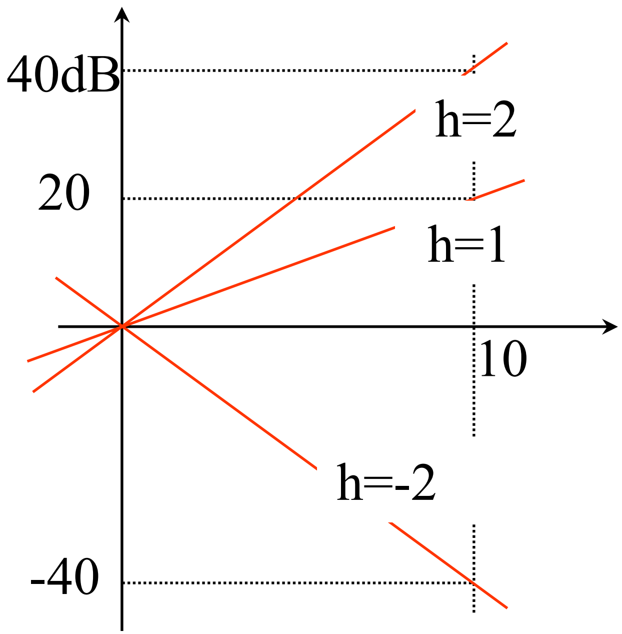
$\left. 100 \right|_{dB} = 40$

Termine monomio $j\omega^h$

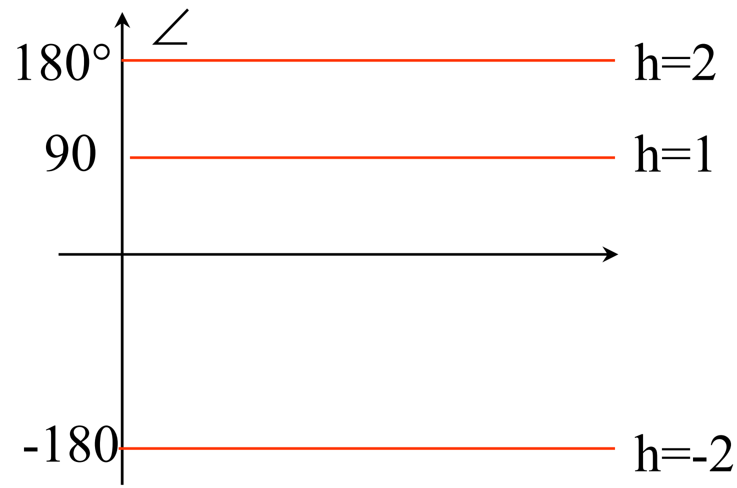
$j\omega^h$



Modulo= 20 h log ω = 20 h λ



Fase = 90° h

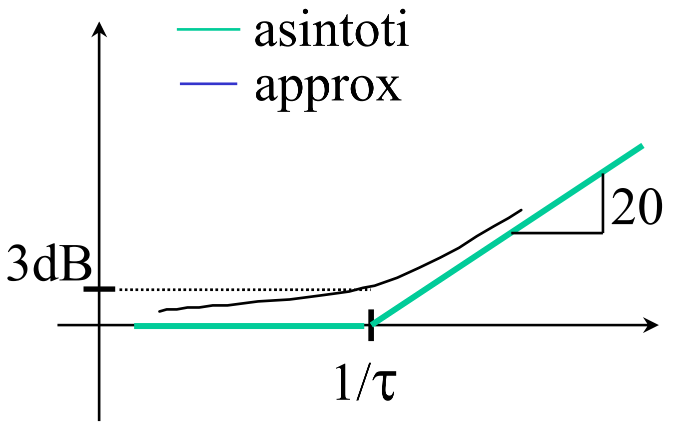


Termine binomio $(1+j\omega\tau)$

(vedi BodeUnPolo_n.vi realizzato in LabView)

$$|\bullet| = 20\log\sqrt{1+\omega^2\tau^2} = 10\log\left(1+\omega^2\tau^2\right)$$

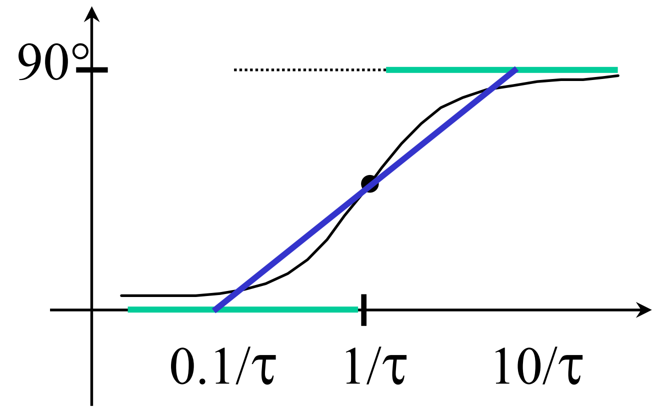
$$\begin{cases} \omega \ll \dfrac{1}{\tau} & 10\log 1 = 0 \\ \omega \gg \dfrac{1}{\tau} & 10\log\omega^2\tau^2 = 20\log\tau + 20\lambda \end{cases}$$



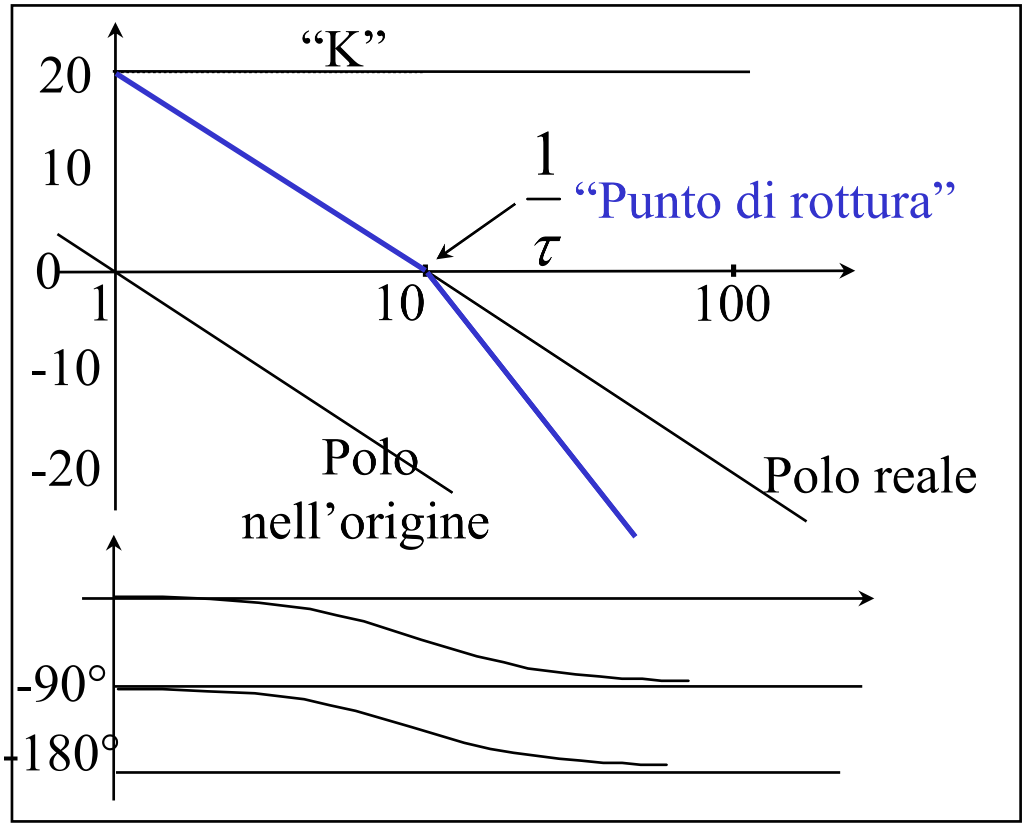
$$|1/G(j\omega)|_{dB} = -|G(j\omega)|_{dB}$$

$$\angle(1+j\omega t) = \tan^{-1}\left(\frac{\omega\tau}{1}\right)$$

$$\begin{cases} \omega \ll \dfrac{1}{\tau} & tg^{-1}(0) = 0 \\ \omega \gg \dfrac{1}{\tau} & tg^{-1}(\infty) = 90^\circ \\ \omega = \dfrac{1}{\tau} & tg^{-1}(1) = 45^\circ \end{cases}$$



$$\angle 1/G(j\omega) = -\angle G(j\omega)$$

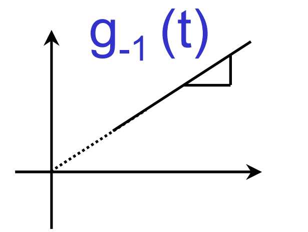


non è K! fattore di guadagno K

- $$G(s) = \frac{100}{s(s+10)} = \frac{10}{s(1+0.1s)}$$

polo nell'origine polo reale

- Significato di K



$$\lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{100}{s(s+10)} = \infty$$

pendenza .. s·G(s)

$$\lim_{s\to 0} s \cdot \frac{1}{s} \cdot s \cdot \frac{100}{s(s+10)} = 10$$

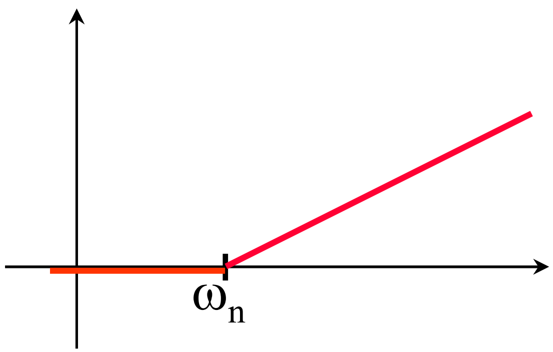
$$W_3(s) = 1 + \frac{2\zeta}{\omega_n} s + \frac{s^2}{\omega_n^2}$$

•Rappresenta *"modi pseudoperiodici"*

- RISONANZE (Denominatore) e
- ANTIRISONANZE (Numeratore)

$$W_3(s)\Big|_{s=j\omega} = \begin{cases} \omega \to 0 & , \ 1 \\ \omega \to \infty & , \ -\dfrac{\omega^2}{\omega_n^2} \end{cases}$$

$$|\ |_{dB} = \begin{cases} \mathbf{0} \\ -40\log\omega_n + 40\lambda \quad (\lambda = \log\omega) \end{cases}$$

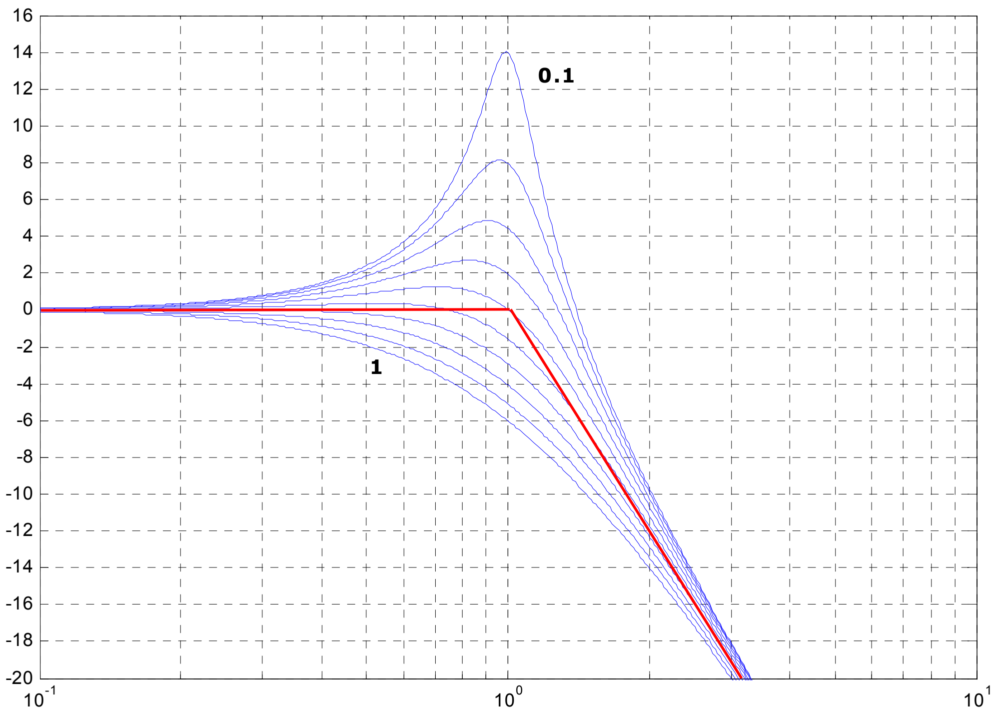


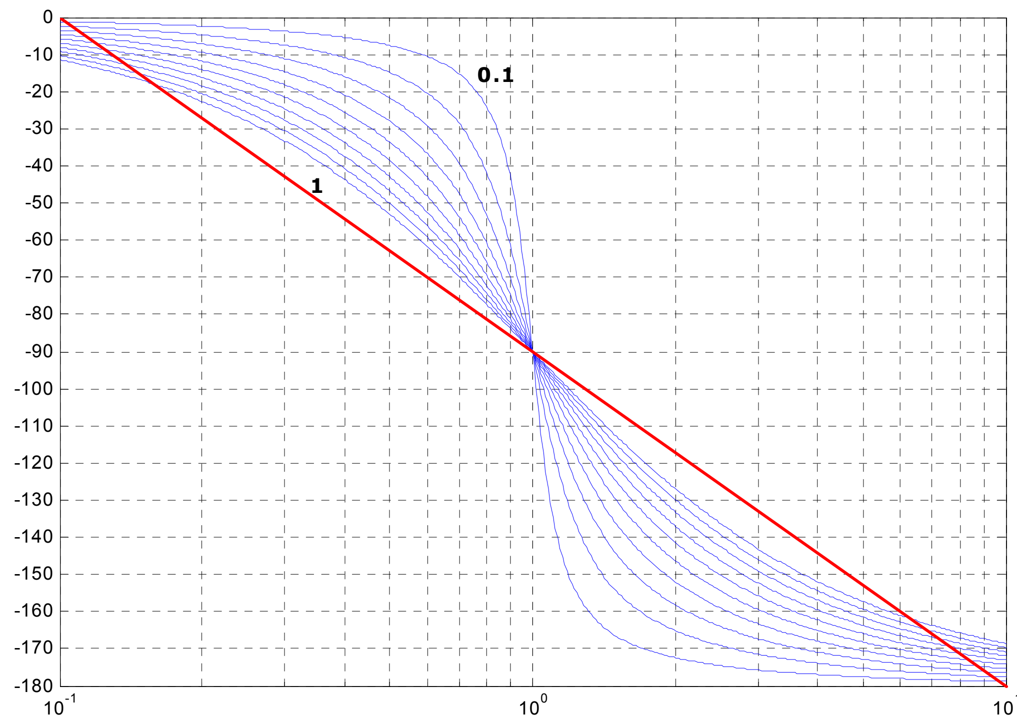
Per $\omega = \omega_n$ $\quad W_3 = 1 + 2j\zeta\frac{\omega_n}{\omega_n} - \frac{\omega_n^2}{\omega_n^2} = 2j\zeta$

$$\begin{cases} |W_3(j\omega_n)| = 2\zeta \qquad (\zeta = 0 \div 1) \\ \angle = +90° \end{cases}$$

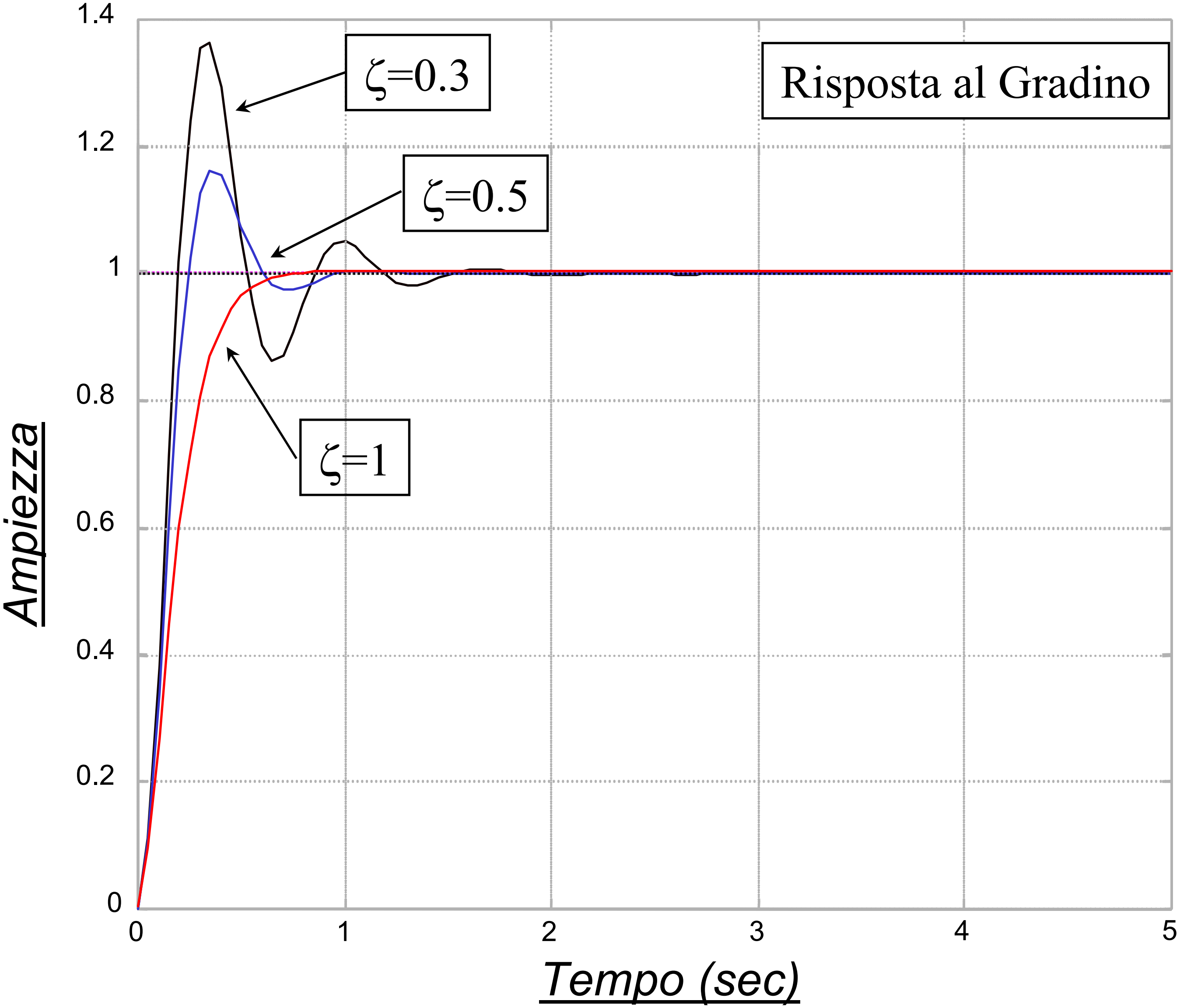
(vedi BodeDuePoli_n.vi realizzato in LabView)

$$\frac{1}{W_3} \qquad \omega_n = 1 \qquad \frac{1}{W_3}$$





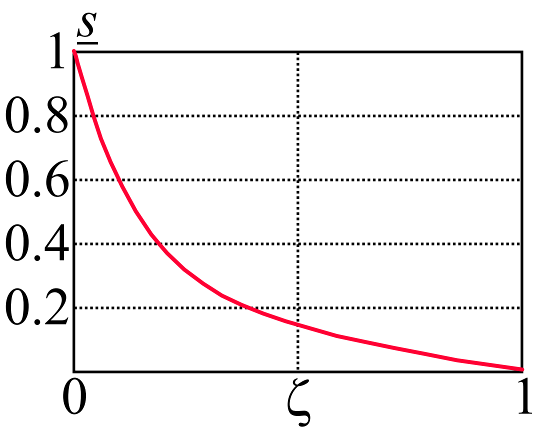
Risposta al gradino del Termine Trinomio

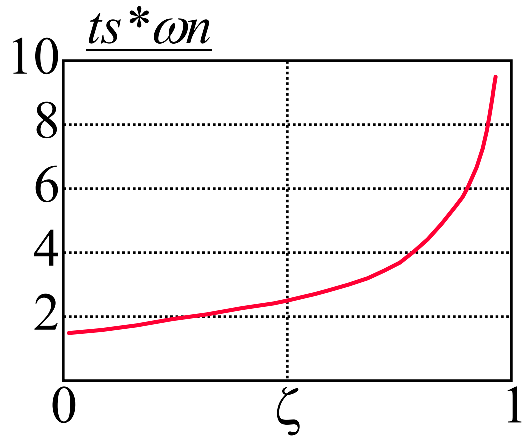


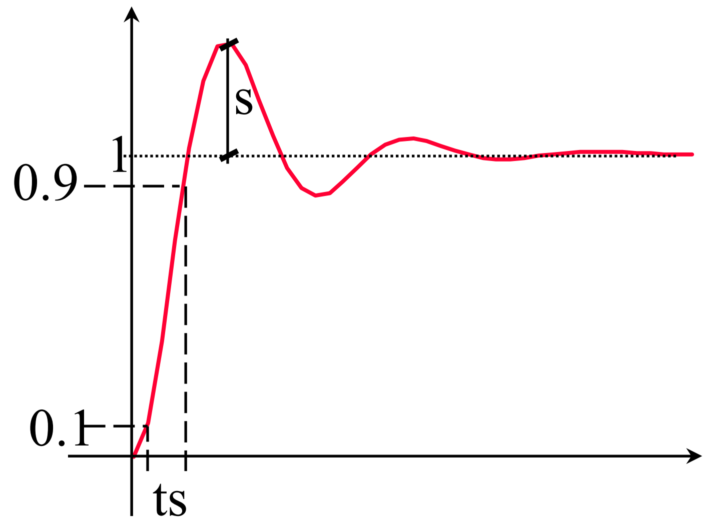
$$L^{-1}\left[\frac{1}{s}\cdot\frac{1}{1+\frac{2\zeta}{\omega_n}s+\frac{s^2}{\omega_n^2}}\right]$$

Parametri del T. Trinomio in funzione di ζ

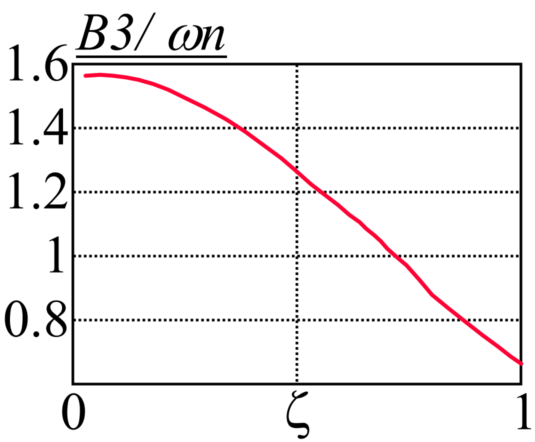
Tempo

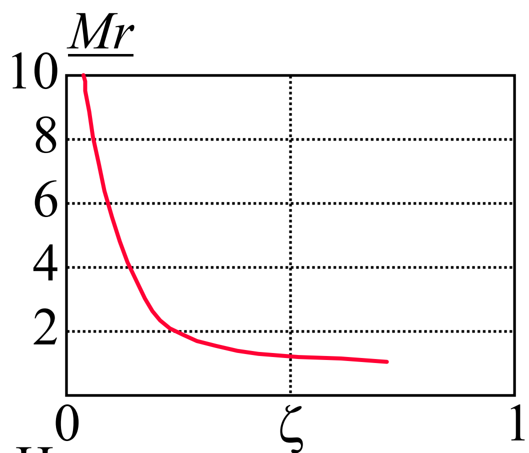


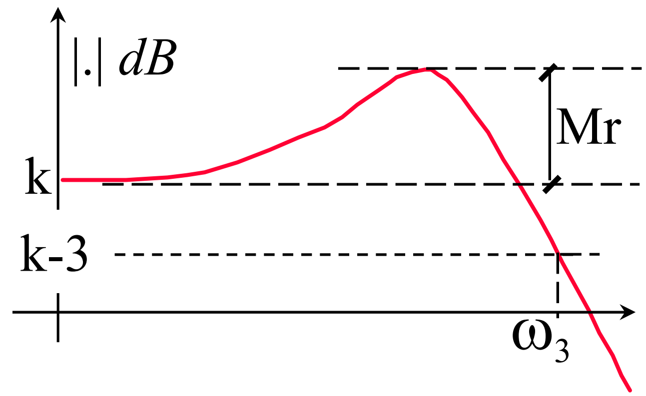




Frequenza







Attenzione :$B_3 = \omega_3 / 2\pi$ in Hz

ω_n in rad/sec

1ª massa: $M\ddot{x}_1 = f(t) - K(x_1 - x_2)$

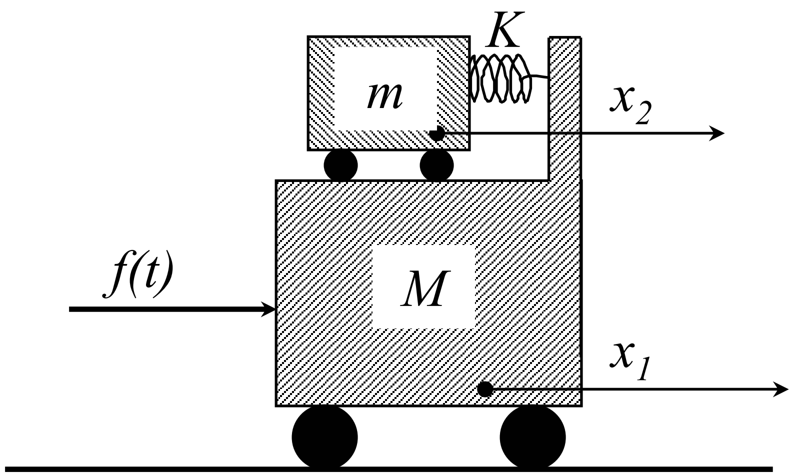
2ª massa: $m\ddot{x}_2 = K(x_1 - x_2)$

Trasformando :

$$\begin{cases} (Ms^2 + K)\cdot X_1(s) = F(s) + K\cdot X_2(s) \\ (ms^2 + K)\cdot X_2(s) = K\cdot X_1(s) \end{cases}$$

$$X_1(s)\left[\left(Ms^2 + K\right) - \frac{K^2}{ms^2 + K}\right] = F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{ms^2 + K}{\left(Ms^2 + K\right)\left(ms^2 + K\right) - K^2} = \frac{ms^2 + K}{\left[mMs^2 + K(m+M)\right]\cdot s^2}$$



Attriti nulli

N.B.

ω di antirisonanza è il punto in cui si annulla il numeratore

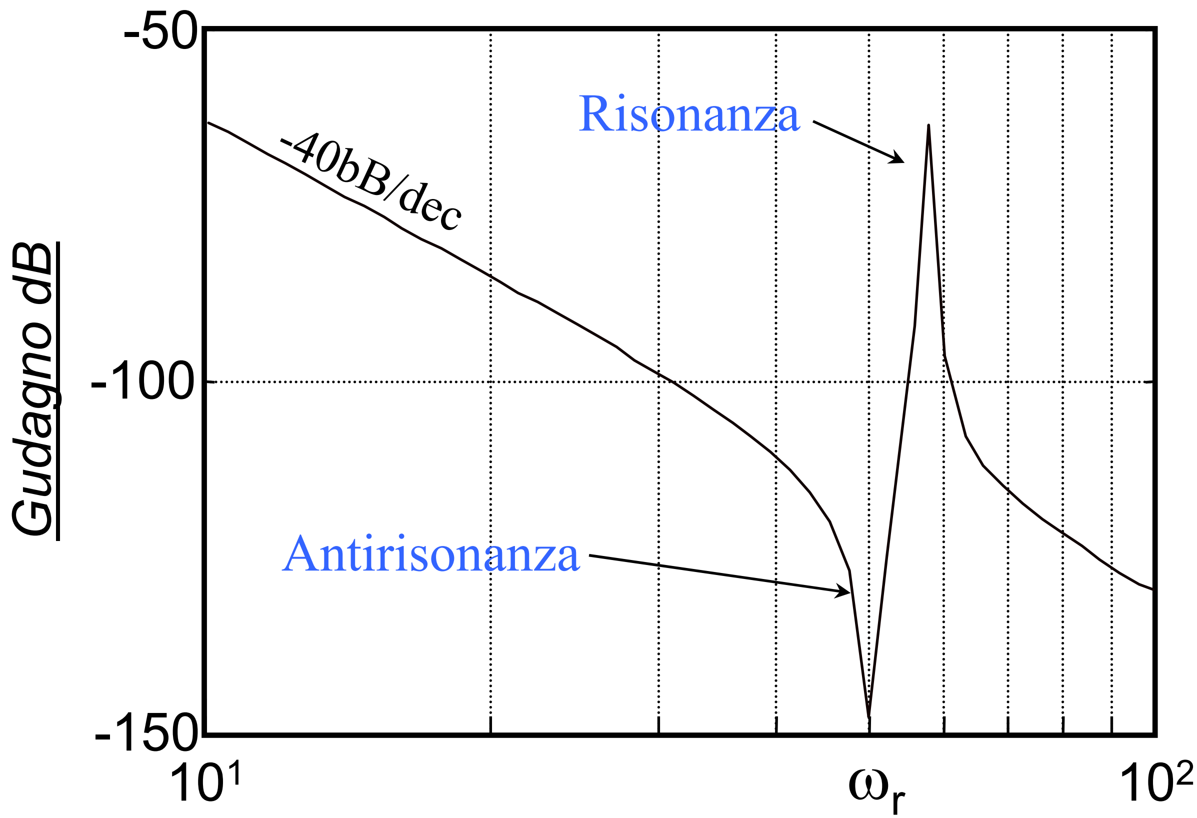
ω di risonanza è il punto in cui si annulla il denominatore

Esempio (continua)

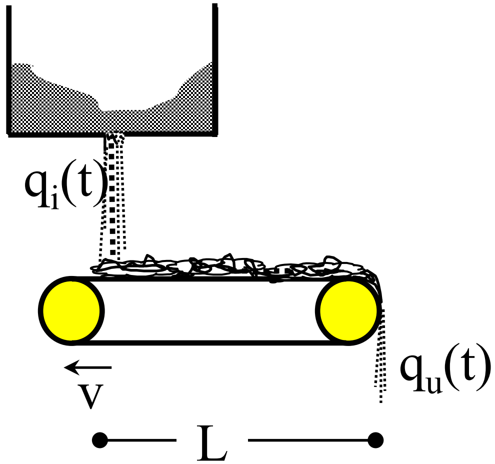
$$Ms^2 + K = 0 \qquad \omega_r = \sqrt{\frac{K}{m}}$$

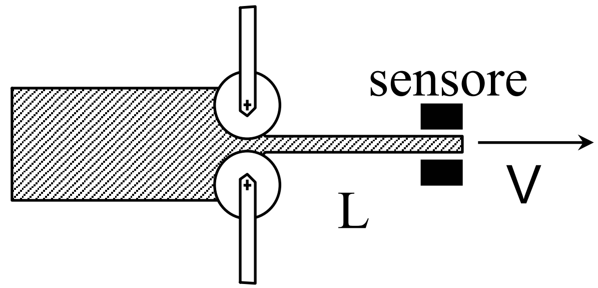
per

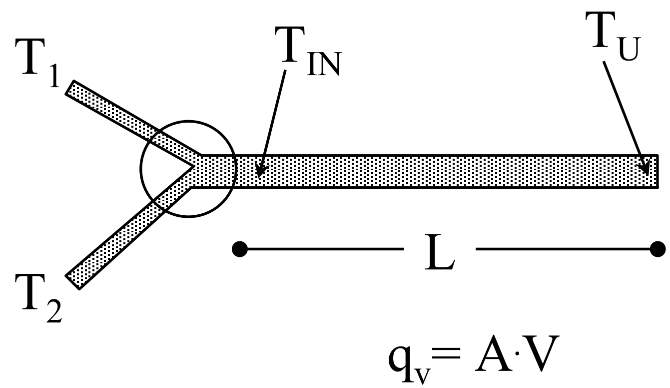
M = 20Kg	$\omega_r = 50$
m = 5Kg	$K = 50^2 \cdot m$



SISTEMI CON RITARDO FINITO (VEDI MARRO PAR.4.7)







$$q_u(t) = q_i\left(t - \frac{L}{V}\right)$$

$$d_{mis}(t) = d\left(t - \frac{L}{V}\right)$$

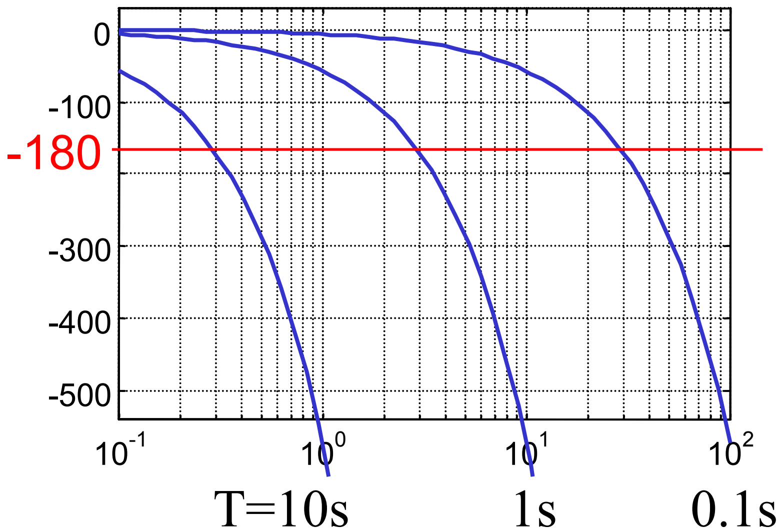
$$T_U(t) = T_{IN}\left(t - \frac{L \cdot A}{q_V}\right)$$

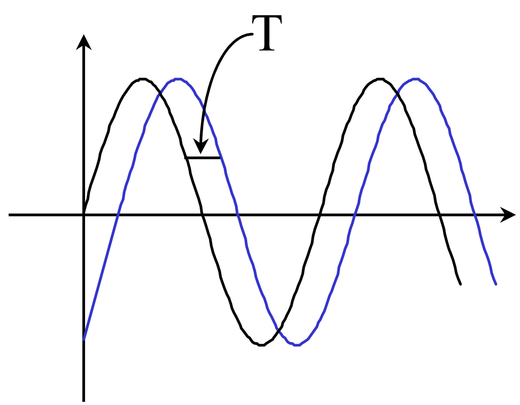
- Teorema della traslazione nel tempo $L[f(t-T)] = e^{-sT}L[f(t)]$ con $T = \text{cost.}$

- Possibili approssimazioni :

$$\begin{cases} e^{-sT} = 1 - sT + \dfrac{s^2T^2}{2} - \dfrac{s^3T^3}{3} + \cdots & \text{(Taylor)} \\ e^{-sT} = \dfrac{1-\dfrac{sT}{2}}{1+\dfrac{sT}{2}} & \text{(Padè)} \end{cases}$$

- Risposta in frequenza $u(t) = \sin(\omega t)$ $y(t) = \sin\left[\omega t - \overbrace{\omega T}^{\varphi}\right]$

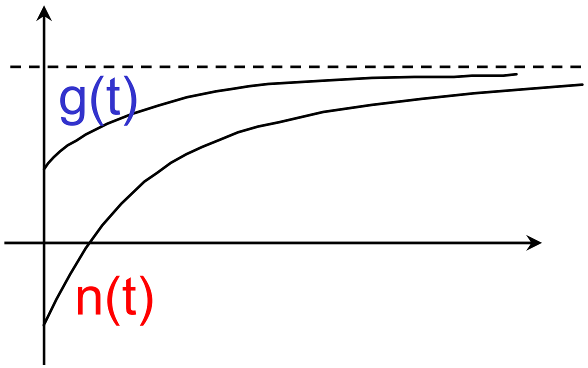




Hanno zeri a parte reale positiva:

$$G(s)=\frac{1+as}{1+bs} \qquad N(s)=\frac{1-as}{1+bs}$$

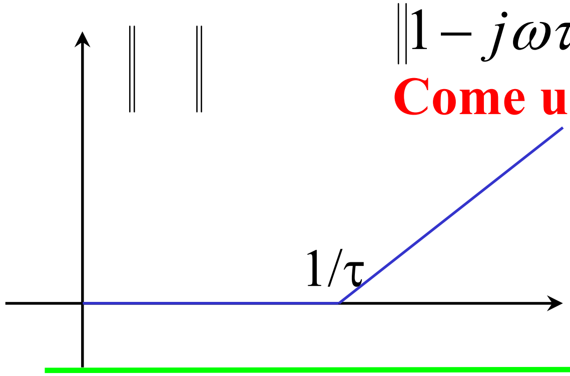
Risposta a gradino



$t=0 \qquad \lim_{s\to\infty} s\cdot\frac{1\pm as}{1+bs}\cdot\frac{1}{s}=\pm\frac{a}{b}$

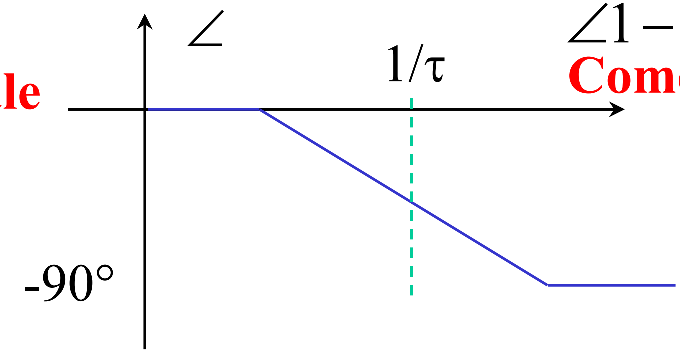
$t=\infty \qquad \lim_{s\to 0} s\cdot\frac{1\pm as}{1+bs}\cdot\frac{1}{s}=1$

Ad uno stimolo positivo rispondono inizialmente con uno spostamento negativo.
Confondono il meccanismo della controreazione !



$$\|1-j\omega\tau\|=\|1+j\omega\tau\|$$

Come uno zero normale



$$\angle 1-j\omega\tau=-\angle 1+j\omega\tau$$

Come un polo