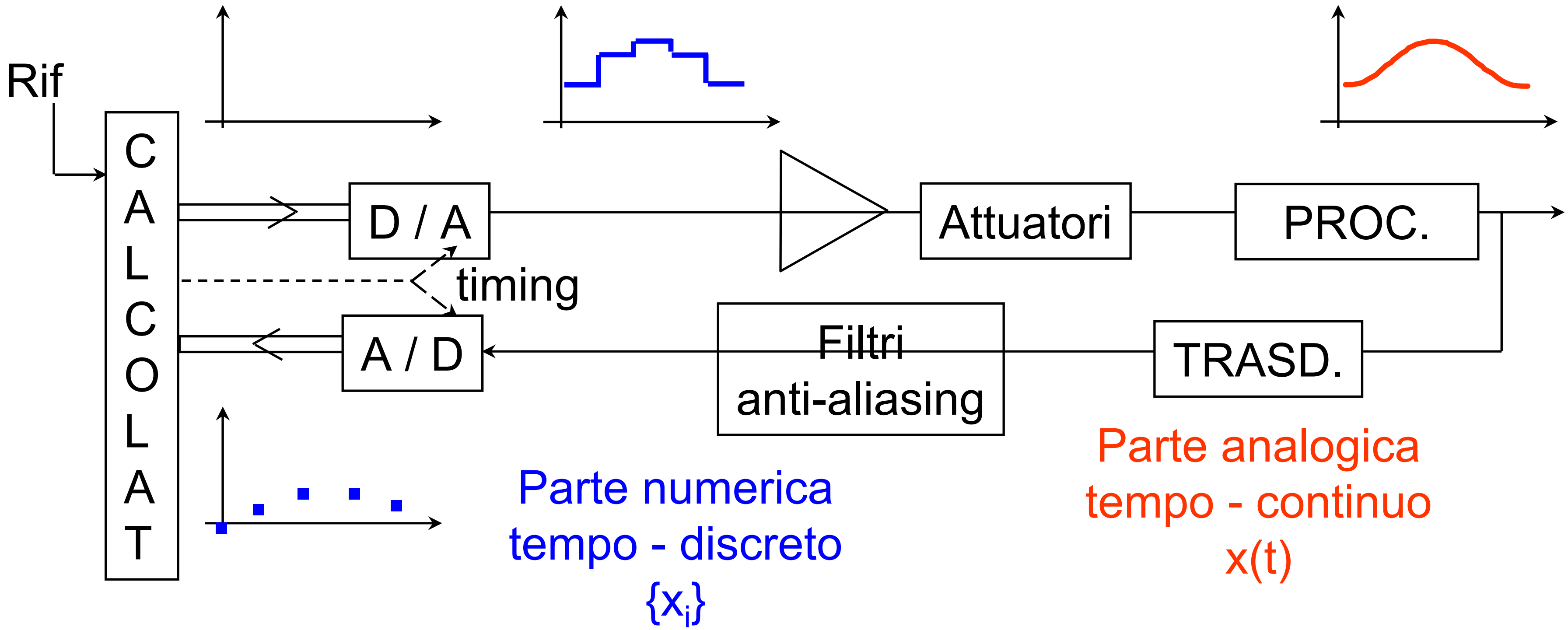
Sistemi a Segnali Campionati (1)

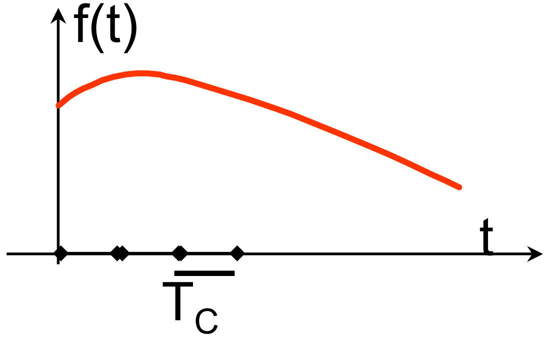
- Controllori a segnali campionari
- Il campionamento
- L’organo di Tenuta
- Spettro di un segnale campionato
- Aliasing

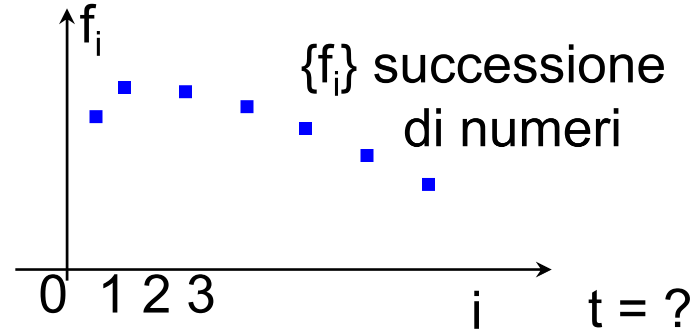


Trovare un modo per rappresentare la parte a tempo discreto che permetta :

a) di utilizzare gli strumenti già noti;

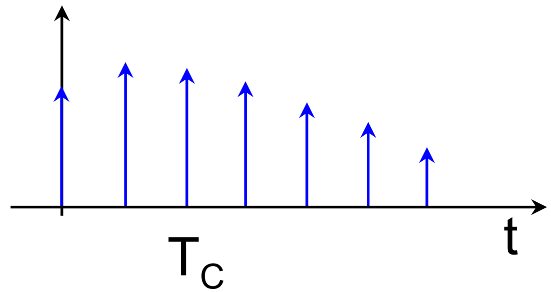
b) di lavorare anche con sistemi misti (tempo continuo/discreto).





Ad essa <u>associamo</u> una successione di impulsi

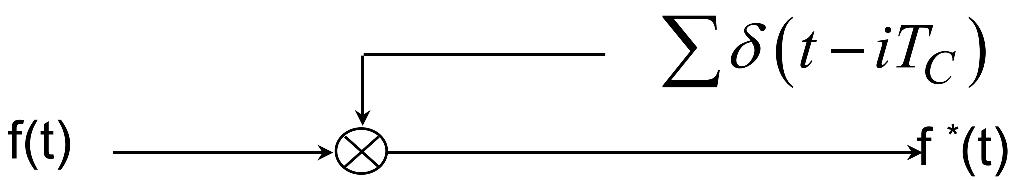
$$f^*(t)=\sum_{i=0}^{\infty} f_i \cdot \delta(t-iT_C)$$



Nella realtà non esiste

In tal modo abbiamo di nuovo un <u>segnale</u> e l'informazione relativa al <u>tempo</u> .

Schema equivalente:



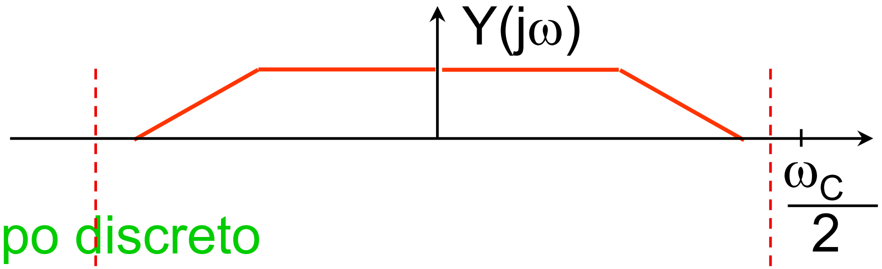
$$F^*(s)=\sum_{i=0}^{\infty} f_i \cdot e^{-iT_C s}$$ Trasformata di Laplace

(Shannon)

Tempo continuo

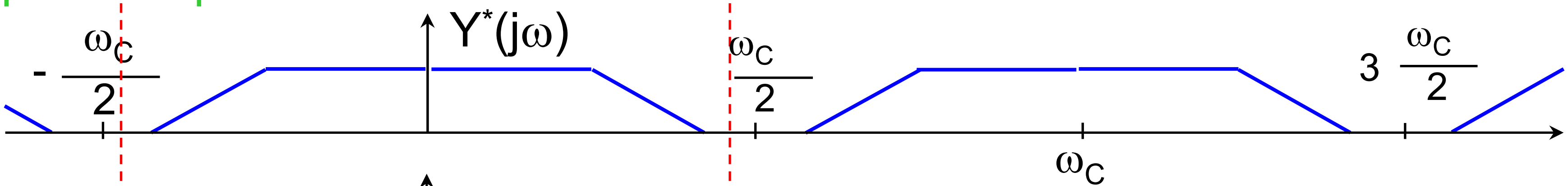
Se $|Y(j\omega)|=0 \;\; \forall\omega>\omega_H$ e $\omega_H \leq \dfrac{\omega_C}{2} = \dfrac{\pi}{T_C}$

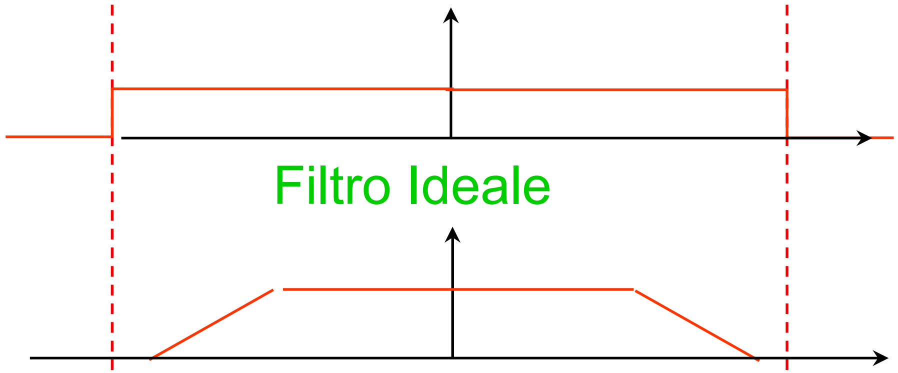
si può ricostruire y(t) da $y^*(t)$



Tempo discreto

Dopo il campionamento...





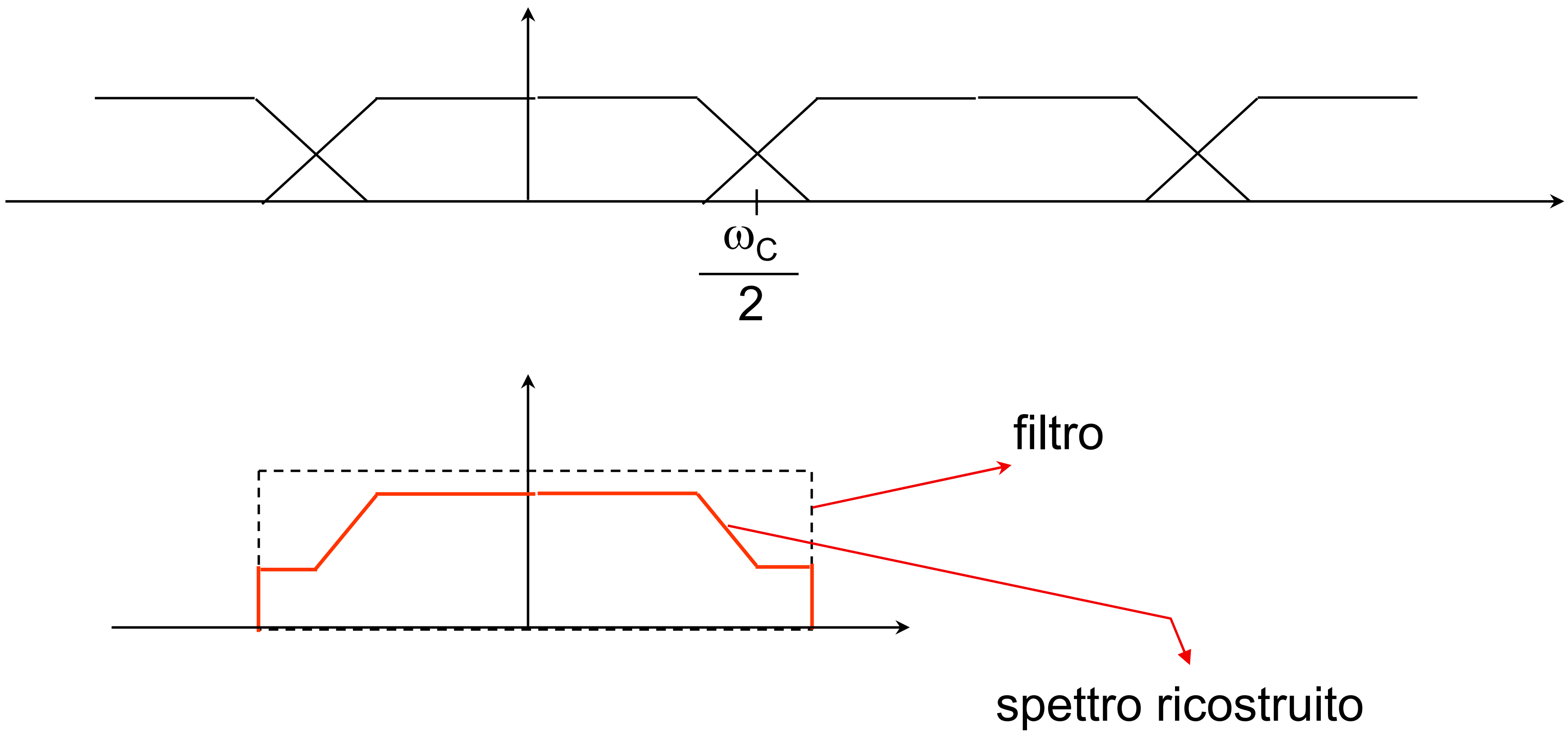
Y ricostruito Tempo continuo

(Filtro passa basso: fa passare le frequenze inferiori di ω^* e blocca perfettamente quelle maggiori di ω^*)

$$\omega_H \leq \omega^* \leq \frac{\omega_C}{2}$$

Cosa accade con Fc troppo bassa

Altrimenti, se $\omega_H > \dfrac{\omega_C}{2}$



I singoli lobi, sommandosi, si modificano,
quindi non si può più isolare quello principale.

Esempio di aliasing

Esempio semplice , T_C = 1 msec

$y_1(t)=\sin(500\pi t)$

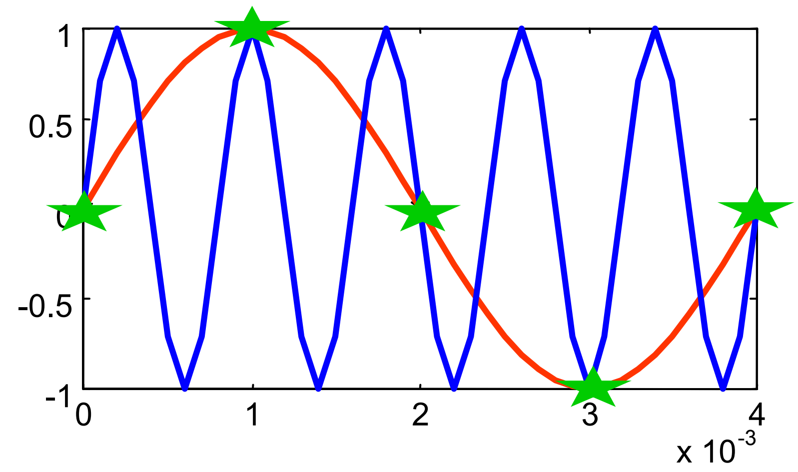
$y_1(iT_C)=\sin(\pi/2 \cdot i)$

i.e. f=250 Hz $\Longrightarrow$ T=4 msec

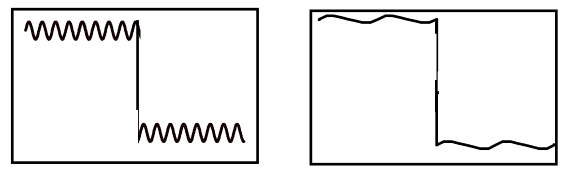
$y_2(t)=\sin(2500\pi t)$

$y_2(iT_C)=\sin[(2\pi+\pi/2)i]=\sin(\pi/2 \cdot i)$

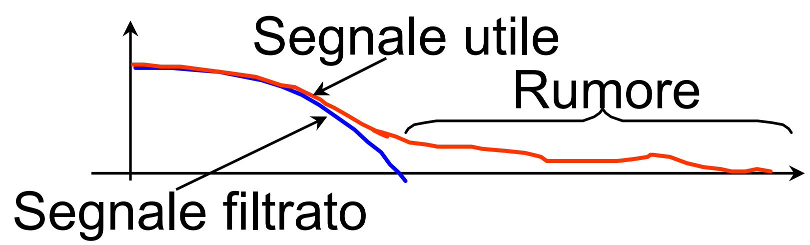
i.e. f=1250 Hz $\Longrightarrow$ T=0.8 msec



Problema reale:
il segnale utile è affetto
da rumore ad alta frequenza



Soluzione:
Usare dei filtri antialiasing
di guardia con $\omega^* << \omega/2$

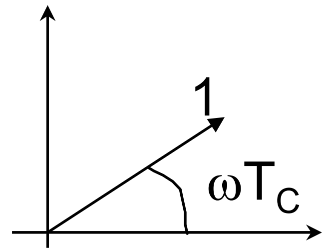


SPETTRO DI UN SEGNALE CAMPIONATO

$$x^*(t) = \sum_{i=0}^{\infty} x(t)\delta(t - iT_C)$$

REM : $L[\delta(t - iT_C)] = e^{-st_i}$ $\qquad (t_i = iT_C)$

$$X^*(s) = L\left[x^*(t)\right] = \sum_{i=0}^{\infty} x(iT_C)e^{-siT_C}$$



REM : $e^{-siT_C} = e^{-\alpha iT_C} \cdot e^{-j\omega iT_C}$ $\qquad s = \alpha + j\omega$

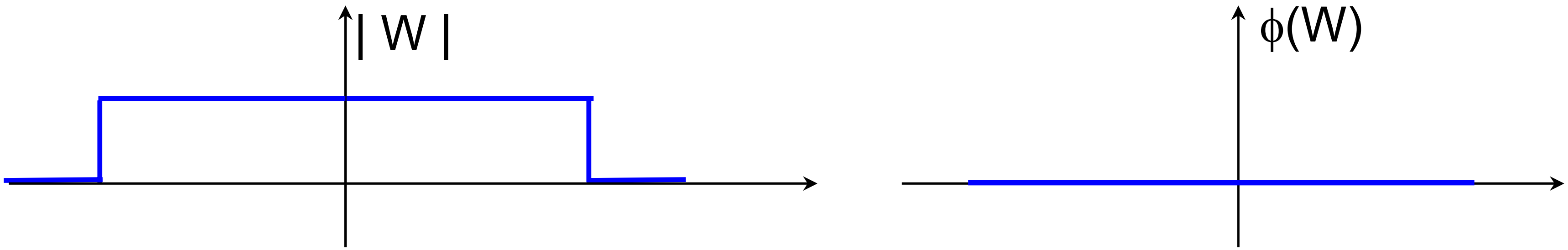
per $\overline{\omega} = \omega + \omega_C = \omega + \dfrac{2\pi}{T_C}$ ⇒ $e^{j\overline{\omega} iT_C} = e^{j\omega iT_C}$

Quindi $X^*(s + kj\omega_C) = X^*(s)$ $\qquad$ K=…,-1,0,1,2,...

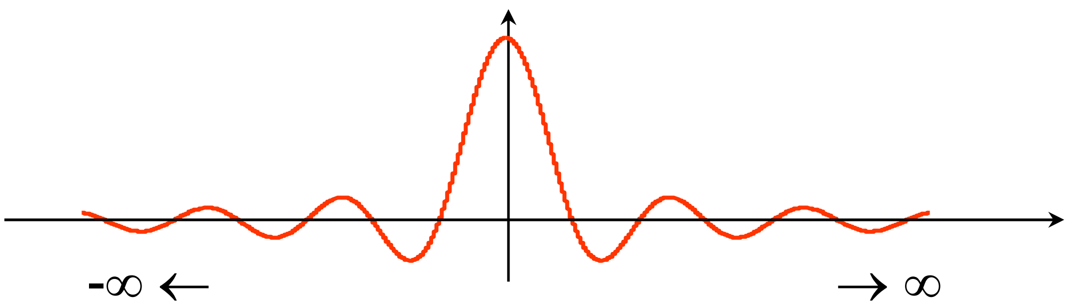
Ovvero $X^*(s)$ è periodica
rispetto a ω=Im[s] con periodo $\qquad \omega_C = \dfrac{2\pi}{T_C}$

Vero anche per $X^*(j\omega) = X^*(s)\Big|_{s = j\omega}$

Antitrasformando la $W(j\omega)$

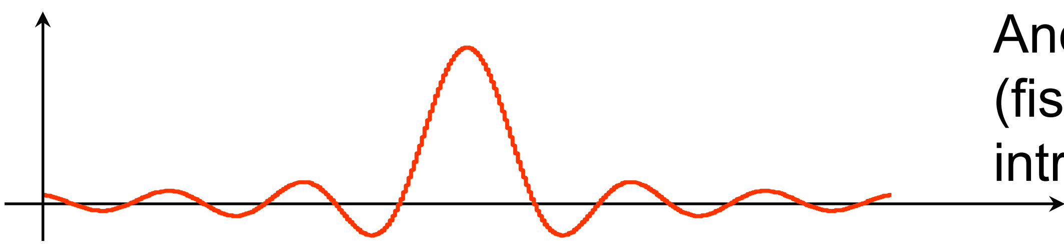


Si ha la risposta impulsiva W(t)=sync(t), non è causale!



$W(t) \neq 0 \quad t = -\infty \cdots\cdots \infty$

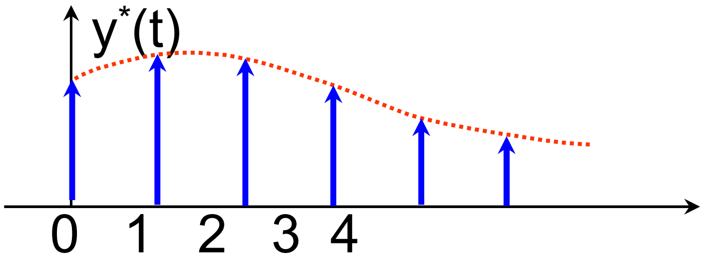
$W(t) \neq 0 \quad t < 0$!!



Anche un suo troncamento e traslazione (fisicamente realizzabile) non va bene: introduce un tempo di ritardo

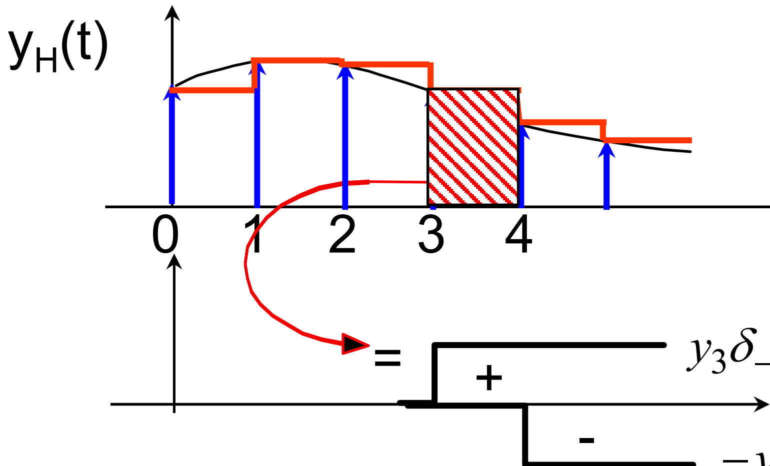
Però questa idea è utilizzabile nelle TLC (e.g. Compact Disk)

Dagli impulsi “ricostruisce” il segnale tempo continuo



$$\mathrm{L}\left[\sum_{0}^{\infty} y_i \delta\left(t - iT_C\right)\right] =$$

$$\underbrace{= \sum_{0}^{\infty} y_i \cdot e^{-isT_C}}_{y^*(s)}$$



$$y_3 \delta_{-1}\left(t - 3T_C\right)$$

$$-y_3 \delta_{-1}\left(t - [3+1]T_C\right)$$

$$Y_H(s) = \sum_{0}^{\infty} y_i \left[\frac{1}{s} e^{-isT_C} - \frac{1}{s} e^{-(i+1)sT_C}\right] =$$

$$= \underbrace{\sum_{0}^{\infty}{}_i \; y_i \cdot e^{-isT_C}}_{y^*(s)} \underbrace{\left(\frac{1 - e^{-sT_C}}{s}\right)}_{T(s)}$$

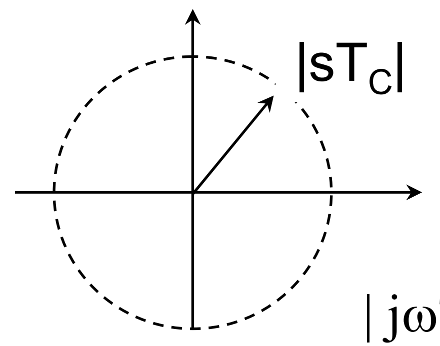
$$T(s) = \frac{1 - e^{-sT_C}}{s}$$

Funzione di trasferimento in s dell’organo di tenuta di ordine 0 (ZOH)

$$T(s)=\frac{1-e^{-sT_C}}{s}=\frac{1}{s}\left(1-\frac{1}{e^{sT_C}}\right)$$

$$e^{x}=1+x+\cancel{\frac{x^2}{2}+\frac{x^3}{6}+\cdots}$$

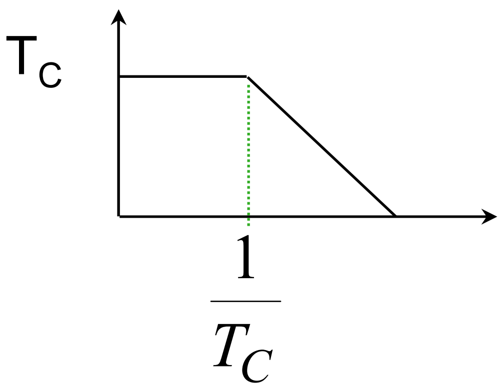
≈ 0 per $|sT_C| << 1$



$|j\omega T_C| << 1$

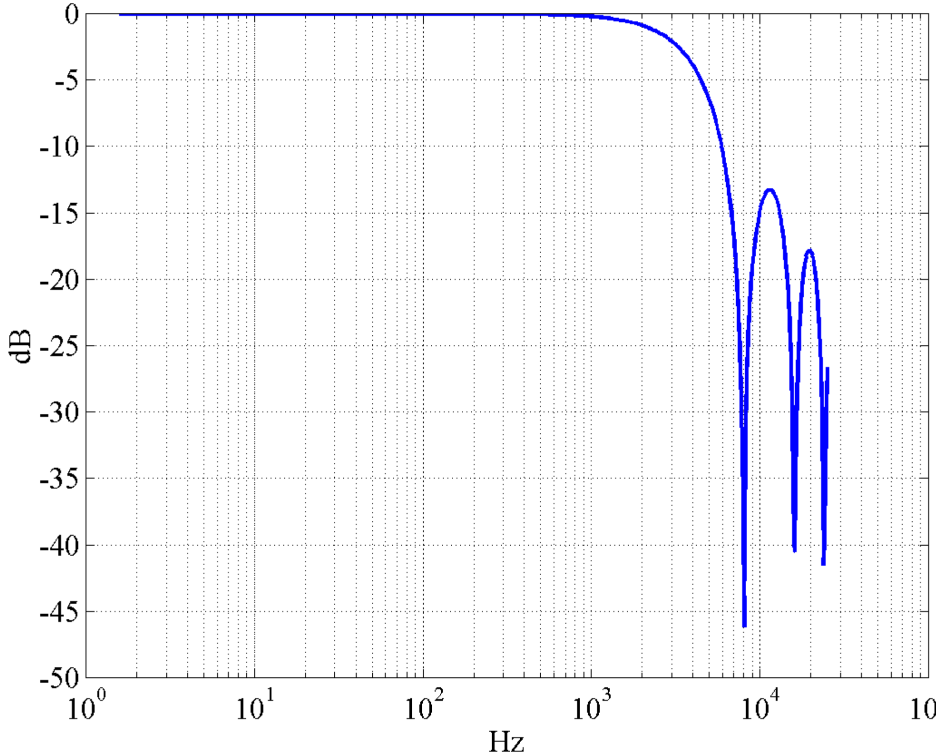
$$\boxed{T_C << \frac{1}{\omega}}$$

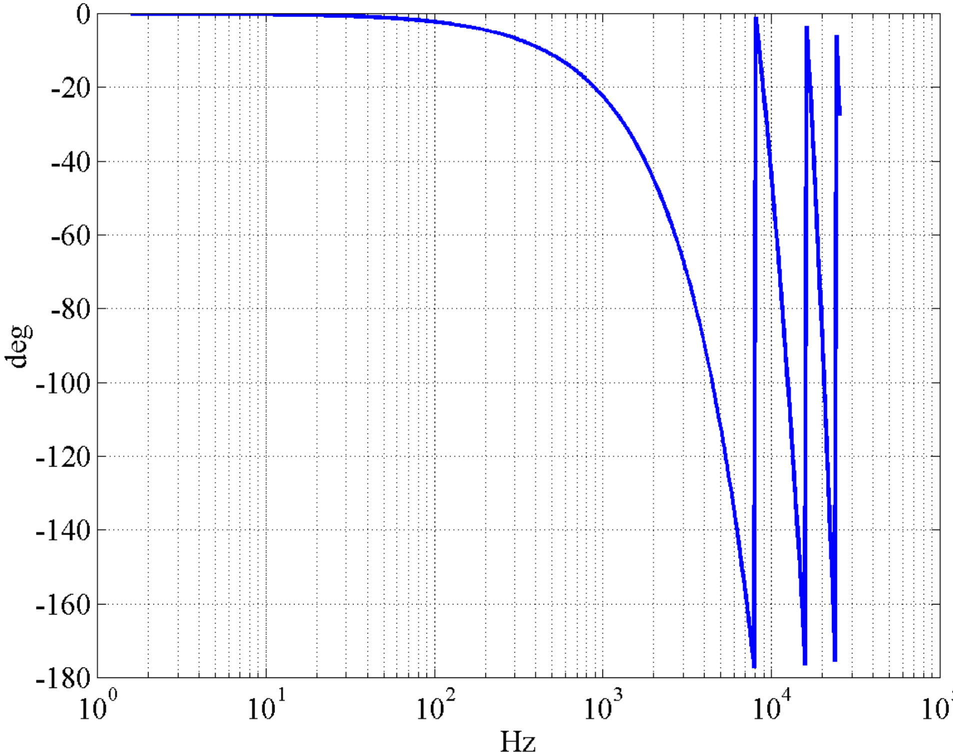
$$T(s)\cong\frac{1}{s}\left(\frac{1+sT_C-1}{1+sT_C}\right)=\frac{T_C}{1+sT_C}$$



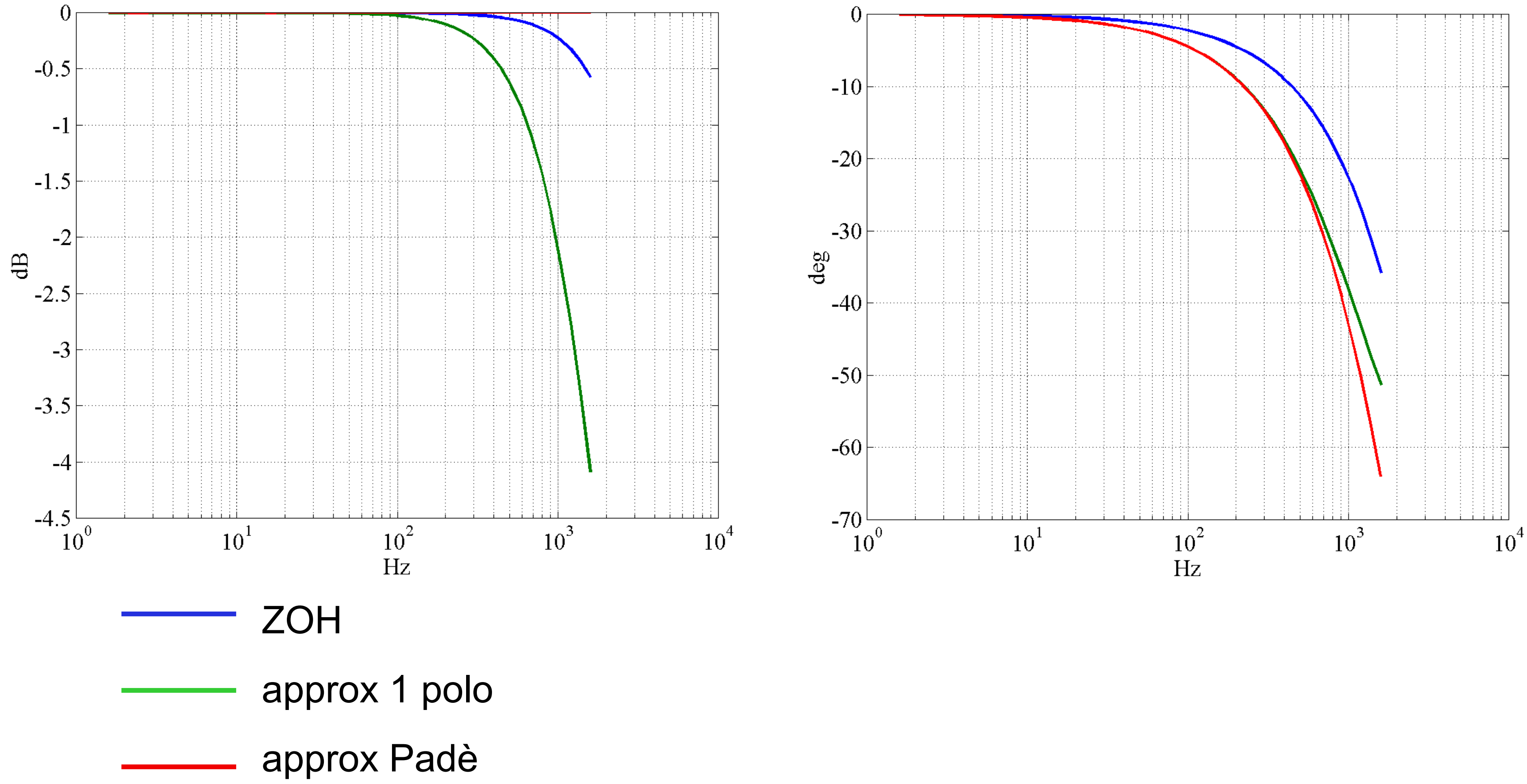
L’organo di tenuta alle basse frequenze ha un comportamento passa basso.

$$F_c = 8000 \text{ Hz}$$

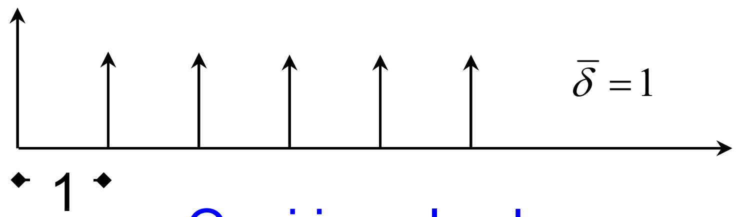


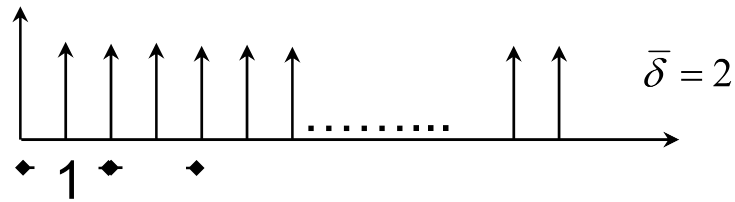


Ha un comportamento passa-basso, quindi ricostruisce ma ha i lobi laterali

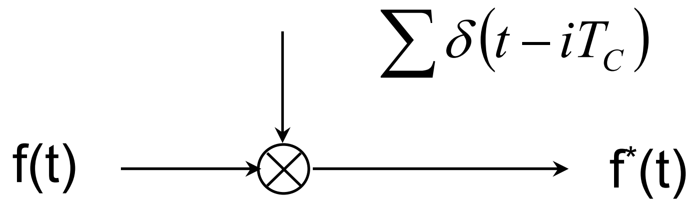

0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
dB
10^0
10^1
10^2
10^3
10^4
Hz
0
-10
-20
-30
-40
-50
-60
-70
deg
Hz
ZOH
approx 1 polo
approx Padè

GUADAGNO DEL CAMPIONAMENTO

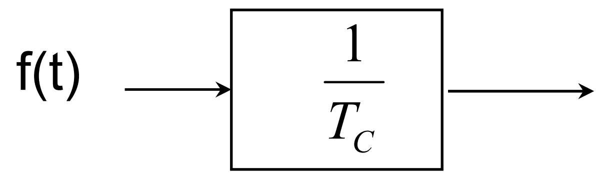




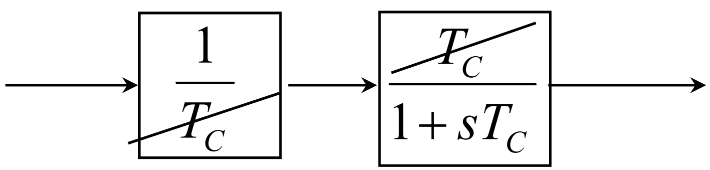
Ogni impulso ha area <u>unitaria</u>



alle basse frequenze: $\sum \delta(t - iT_C) \approx \bar{\delta}$



quindi consideriamo campionamento + tenuta



$\approx$ (basse frequenze)