Trasformata di Fourier e Legame con la Trasformata di Laplace

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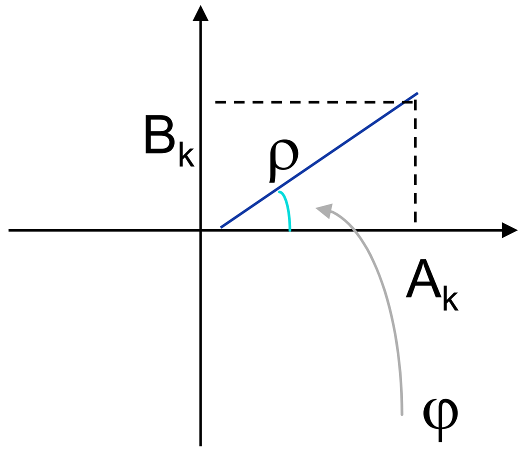
Trasformata di Fourier
Trasformata di Laplace
Confronto in Maple

$$f(t) = F(t \pm KT) \ \text{periodica}, \quad \text{T : periodo}$$

$$f(t) = \frac{A_0}{2} + \sum_{R=1}^{\infty} \left[A_k \cos \frac{2\pi k}{T} t + B_k \sin \frac{2\pi k}{T} t \right]$$



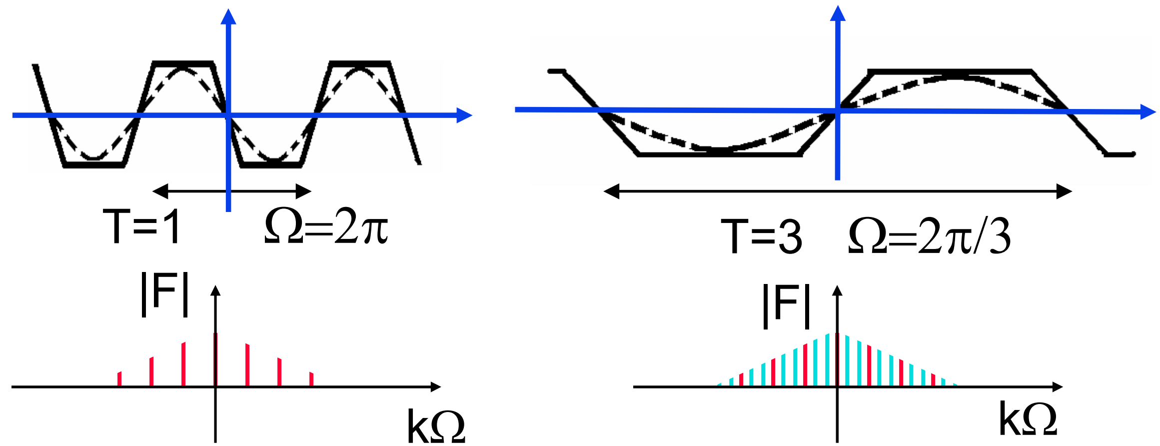
$$F(k\Omega) = \rho\, e^{j\varphi}$$

- $$\frac{A_0}{2} = \frac{1}{T} \int_{-T/2}^{T/2} f(t)dt \ \left[valore \ \ medio \right]$$
- $$A_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t)\cos \frac{2\pi k}{T} t \ dt;$$
- $$B_k = \frac{2}{T} \int_{-T/2}^{T/2} f(t)\sin \frac{2\pi k}{T} t \ dt$$

utilizzando i numeri complessi

$$\left[\begin{array}{l} f(t) = \dfrac{\Omega}{2\pi} \displaystyle\sum_{-\infty}^{\infty} {}_k F(k\Omega) e^{jk\Omega t} \\ F(k\Omega) = \dfrac{1}{T} \displaystyle\int_{-T/2}^{T/2} f(t) e^{-jk\Omega t} dt \end{array} \right] \qquad \Omega = \frac{2\pi}{T}$$

La serie è per segnali periodici, un segnale qualsiasi può essere visto come periodico con periodo infinito.



$$\left[\begin{array}{l} F(k\Omega) = \dfrac{1}{T}\displaystyle\int_{-T/2}^{T/2} f(t)e^{-jk\Omega t}\,dt \\[2ex] f(t) = \dfrac{\Omega}{2\pi}\displaystyle\sum_{-\infty}^{\infty}{}_k F(k\Omega)e^{jk\Omega t} \end{array}\right] \qquad \Omega = \frac{2\pi}{T}$$

$$T \to \infty,\ \ \Omega \to 0,\ \ k\Omega \to \omega,\ \ \sum \to \int \quad ➔$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\,dt$$

$$F(t) = \int_{\infty}^{\infty} F(\omega)\, e^{j\omega t}\,d\omega$$

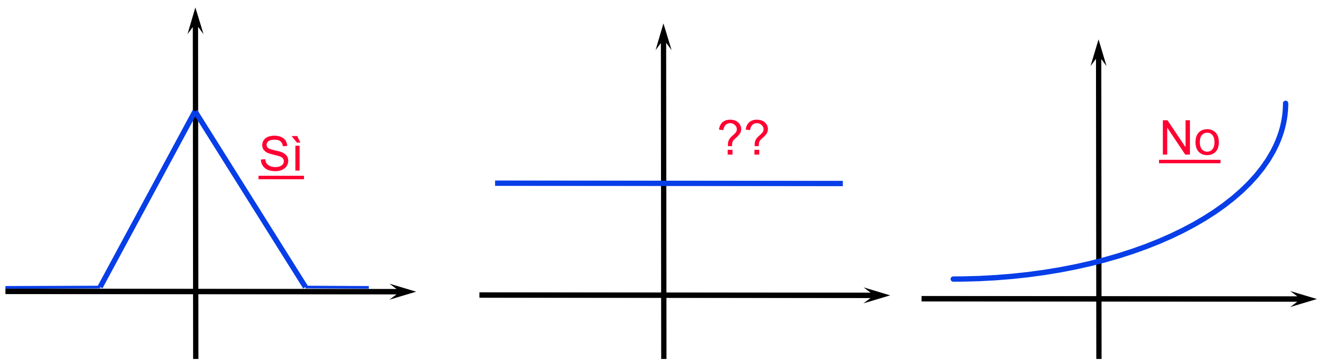
Condizione: f(t) sommabile

$$\int_{-\infty}^{\infty} |f(t)|\,dt < \infty$$

$$\left.\begin{aligned} F(\omega) &= \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt \\ F(t) &= \int_{\infty}^{\infty} F(\omega)\, e^{j\omega t} d\omega \end{aligned}\right\}$$

Corrispondenza biunivoca

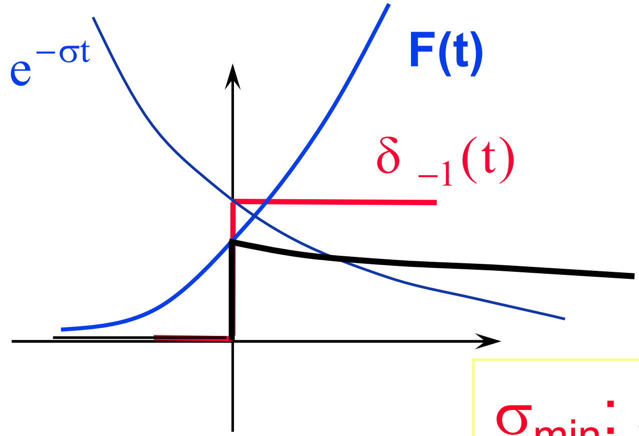
$$solo \quad se \quad \int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$



Sicchè proprio i casi più interessanti creano problemi

"TRUCCO" Invece di trasformare *f(t)*, trasformiamo:

$$\delta_{-1}(t)f(t)e^{-\sigma t}$$



σ_{min}: ascissa di convergenza

$$Pongo \quad s = \sigma + j\omega$$

$$F(\sigma + j\omega) = \int_0^\infty f(t)e^{-\sigma t} \cdot e^{-j\omega t} dt$$

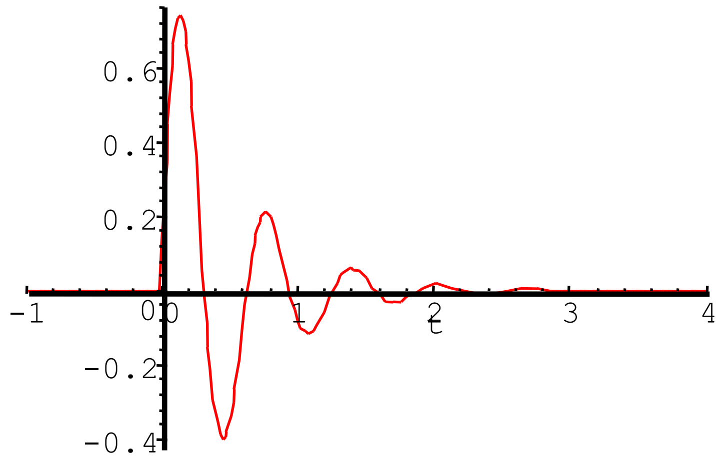
$$F(s) = \int_0^\infty f(t)e^{-st} dt$$

Trasformata di LAPLACE

```
> readlib(fourier):
> sin(10*t)*exp(-2*t)*Heaviside(t);
```

$$\sin(10\,t)\, e^{(-2\,t)}\, \mathrm{Heaviside}(t)$$

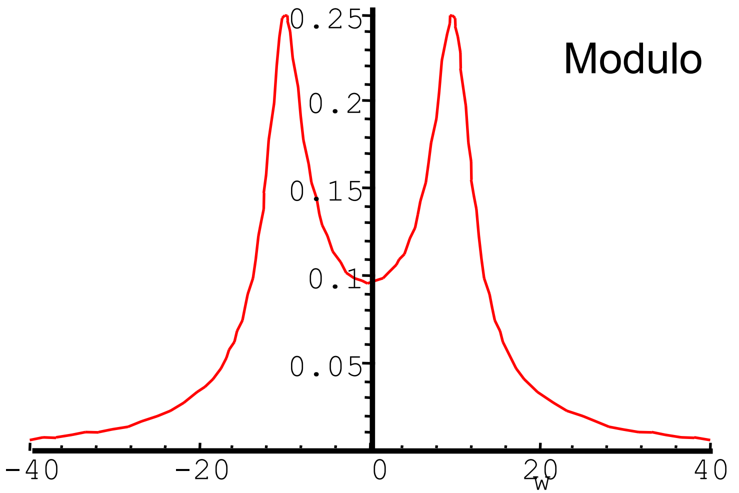
```
> plot(sin(10*t)*exp(-2*t)*Heaviside(t),t=-1..4, color= 'red');
```




```
> F:= fourier(sin(10*t)*exp(-2*t)*Heaviside(t),t,w);
```

$$F := -\frac{1}{2}\,\frac{I}{2 + I\,w - 10\,I} + \frac{1}{2}\,\frac{I}{2 + I\,w + 10\,I}$$

```
> plot(abs(F), w=-40..40, color= 'red');
```



Trasformata di Laplace in Maple

> F:= laplace(exp(-2*t)*sin(10*t),t,s);

$$10 \frac{1}{s^2 + 4 s + 104}$$

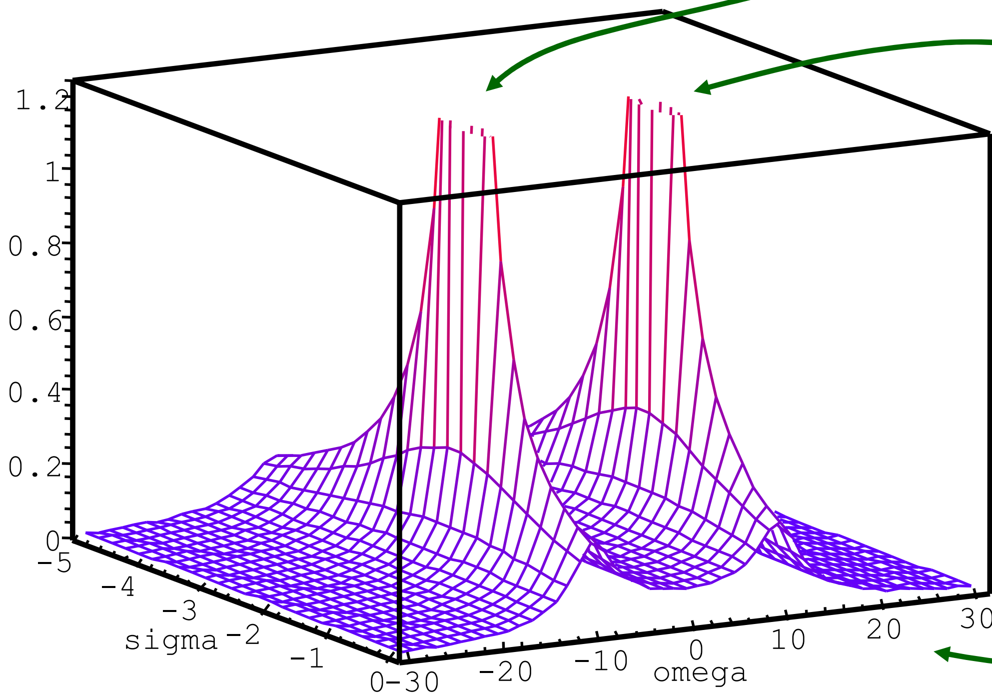
> F1:= subs(s=sigma+I*omega,F);

$$F1 := 10 \frac{1}{(\sigma + I\,\omega + 2)^2 + 100}$$

> plot3d(abs(F1), sigma=-5..0,omega=-30..30, view=0..1.25);

Grafico del Modulo

(fase non riportata)



Effetto dei poli della F(s). Il modulo va all'infinito

La sezione su questo piano è la risposta armonica (confronta con Fourier)

> Poli:= solve(denom(F)=0);

$$Poli := -2 + 10\,I,\ -2 - 10\,I$$